Modeling moderate and extreme urban precipitation at high spatio-temporal resolution

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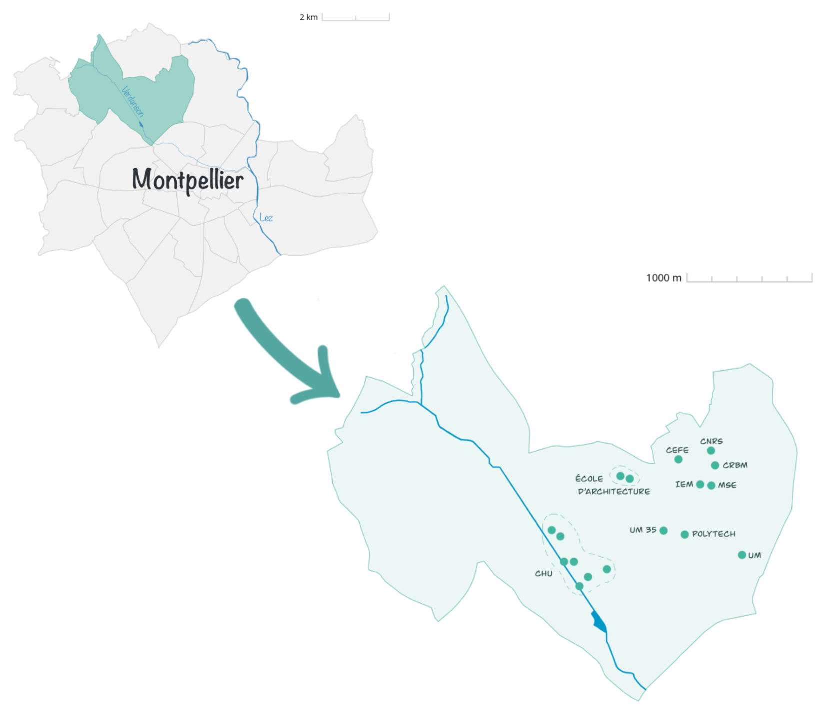
Abstract

- Fine spatial and temporal scale
- Univariate modeling of moderate and intense rainfall with EGPD
- Spatio-temporal dependence modeling with weighted least squares estimation
- Brown-Resnick dependence
- Relax separability assumption with advection consideration

Montpellier, South of France

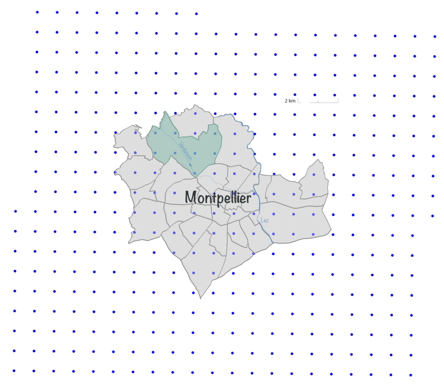
- Mediterranean episodes, localized rainfall
- Urban area, flood risks
- Collaboration with hydrologists

Data [1]



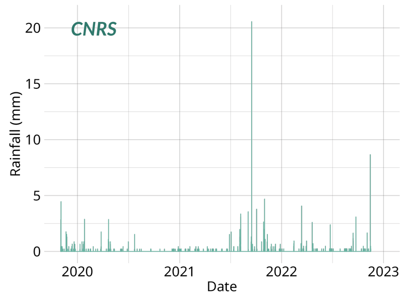
$\mathcal{S}_{\text{stations}} = \{17$ rain gauge locations$\}$

- Period: $[2019, 2022]$
- High temporal resolution: Every minute ⟶ 5-minute aggregation
- Small spatial scale: Interdistance $\in [77, 1531]$ meters
- Other dataset: Hourly COMEPHORE data with a 1 km^2-resolution over Montpellier [2]



$\mathcal{S}_{\text{grid}} = \{400$ pixels$\}$

Let $\mathcal{S} \subset \mathbb{R}^2$ be our spatial domain and let $\mathcal{T} \subset \mathbb{R}_+$ be our temporal domain with equidistant time points. Let $\Lambda_{\mathcal{S}} \subset \mathbb{R}^2$ and $\Lambda_{\mathcal{T}} \subset \mathbb{R}_+$ be sets of spatial and temporal lags respectively.

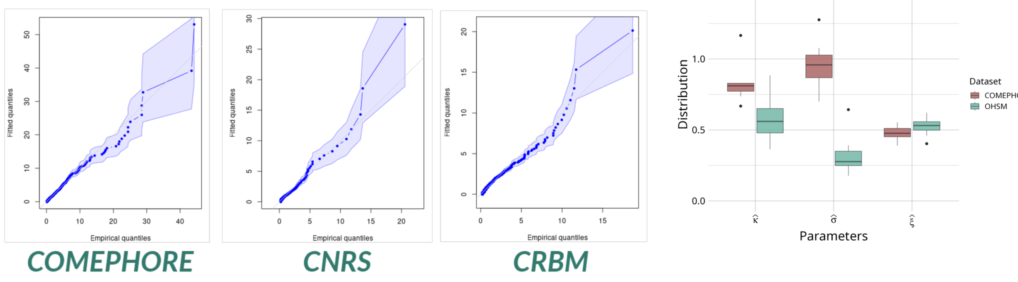


Moderate

Extreme

Univariate

Rainfall measurement: $X_{\boldsymbol{s}}$ at a given site $\boldsymbol{s} \in \mathcal{S}$.



EGPD estimates

Extended GPD [3]

$$F(x) = G\left(H_\xi\left(\frac{x}{\sigma}\right)\right), \text{ with } G(y) = y^\kappa,\ \kappa > 0$$

GPD

$$X_s - u \mid X_s > u \to H_\xi \text{ with } H_\xi\left(\frac{x-u}{\sigma}\right) = \begin{cases} 1 - \left(1 + \xi\frac{x-u}{\sigma}\right)_+^{-1/\xi} & \text{if } \xi \neq 0, \\ 1 - e^{-\frac{x-u}{\sigma}} & \text{if } \xi = 0, \end{cases}$$

where $a_+ = \max(a, 0)$, $\sigma > 0$, $x - u > 0$

Dependence

Rainfall field: $\boldsymbol{X} = \{X_{\boldsymbol{s},t},\ (\boldsymbol{s}, t) \in \mathcal{S} \times \mathcal{T}\}$ a stationary and isotropic process with a Brown-Resnick dependence [4].

- **Extreme dependence** determined by the **spatio-temporal extremogram**:

$$\chi(\boldsymbol{h}, \tau) = \lim_{q \to 1} \chi_q(\boldsymbol{h}, \tau), \quad \text{with} \quad \chi_q(\boldsymbol{h}, \tau) = \mathbb{P}(X^*_{\boldsymbol{s},t} > q \mid X^*_{\boldsymbol{s}+\boldsymbol{h},t+\tau} > q),\ \boldsymbol{h} \in \Lambda_{\mathcal{S}}, \tau \in \Lambda_{\mathcal{T}}$$

with $q \in [0, 1[$ and $X^*_{\boldsymbol{s},t}$ the standardized univariate margins.

- **Dependence** determined by the **spatio-temporal variogram** of $\boldsymbol{W}$:

$$\gamma(\boldsymbol{h}, \tau) = \frac{1}{2} Var\left(W_{\boldsymbol{s},t} - W_{\boldsymbol{s}+\boldsymbol{h},t+\tau}\right),\ \boldsymbol{h} \in \Lambda_{\mathcal{S}}, \tau \in \Lambda_{\mathcal{T}}$$

r-Pareto

For all $\boldsymbol{s} \in \mathcal{S}$ and $t \in \mathcal{T}$,

$$u^{-1} X^*_{s,t} \mid X^*_{s_0,t_0} > u \xrightarrow[u \to \infty]{d} Z_{s,t},$$

and $Z_{s,t} = R e^{W_{s,t} - W_{s_0,t_0} - \gamma(s - s_0, t - t_0)}$, with $(\boldsymbol{s}_0, t_0)$ a given space-time location, $R \sim Pareto(1)$ and u a threshold.

Max-stable

For all $\boldsymbol{s} \in \mathcal{S}$ and $t \in \mathcal{T}$,

$$X_{\boldsymbol{s},t} = \bigvee_{j=1}^{\infty} \xi_j e^{W^{(j)}_{s,t} - \gamma(s,t)}$$

- $(\xi_j)_{j \geq 1}$: Poisson process with intensity $\xi^{-2} d\xi$
- $W^{(j)}$: indep. rep. of a Gaussian random field $\boldsymbol{W}$
- γ: spatio-temporal variogram of $\boldsymbol{W}$

Link extremogram-variogram and separabilty [5]

Assumption of additive separability: $\frac{1}{2}\gamma(\boldsymbol{h}, \tau) = \beta_1 \|\boldsymbol{h}\|^{\alpha_1} + \beta_2 \tau^{\alpha_2},\ 0 < \alpha_1, \alpha_2 \leq 2,\ \beta_1, \beta_2 > 0$

With ϕ the std normal cdf,

Spatio-temporal

$$\chi(\boldsymbol{h}, \tau) = 2\left(1 - \phi\left(\sqrt{\tfrac{1}{2}\gamma(\boldsymbol{h}, \tau)}\right)\right)$$

Transformation:

$$\boldsymbol{\eta(\chi) = 2\log\left(\phi^{-1}\left(1 - \tfrac{1}{2}\chi\right)\right)}$$

Spatial

$$\eta\left(\chi(\boldsymbol{h}, 0)\right) = \log \beta_1 + \alpha_1 \log\|\boldsymbol{h}\| =: c_1 + \alpha_1 x_{\boldsymbol{h}}$$

Temporal

$$\eta\left(\chi(0, \tau)\right) = \log \beta_2 + \alpha_2 \log \tau =: c_2 + \alpha_2 x_\tau$$

Weighted Least Squares Estimation (WLSE)

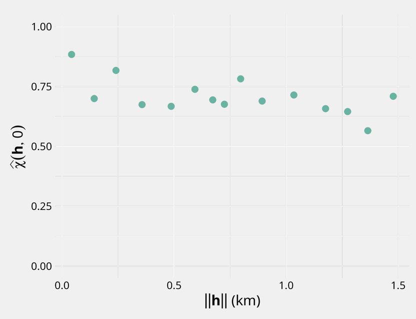
$$\begin{pmatrix} \widehat{c_i} \\ \widehat{\alpha_i} \end{pmatrix} = \operatorname{argmin}_{c_i, \alpha_i} \sum_x w_x \left(\eta\left(\widehat{\chi}\right) - (c_i + \alpha_i x)\right)^2$$

Spatial dependence

Extremogram estimator: For a fixed $t \in \mathcal{T}$ and q a high quantile,

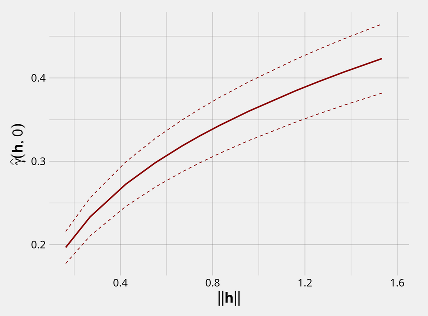
$$\widehat{\chi}_q^{(t)}(\boldsymbol{h}, 0) = \frac{\frac{1}{|N_{\boldsymbol{h}}|}\sum_{i,j \mid (s_i, s_j) \in N_{\boldsymbol{h}}} \mathbb{1}_{\{X^*_{s_i,t} > q, X^*_{s_j,t} > q\}}}{\frac{1}{|\mathcal{S}|}\sum_{i=1}^{|\mathcal{S}|} \mathbb{1}_{\{X^*_{s_i,t} > q\}}}$$

where $C_{\boldsymbol{h}}$ are equifrequent distance classes and $N_{\boldsymbol{h}} = \left\{(\boldsymbol{s}_i, \boldsymbol{s}_j) \in \mathcal{S}^2 \mid \|\boldsymbol{s}_i - \boldsymbol{s}_j\| \in C_{\boldsymbol{h}}\right\}$.



Empirical extremogram with $q = 95\%$

Transformation and WLSE ⟶



Spatial variogram estimate $\widehat{\gamma}(\boldsymbol{h}, 0) = 2\widehat{\beta}_1 \|\boldsymbol{h}\|^{\widehat{\alpha}_1}$

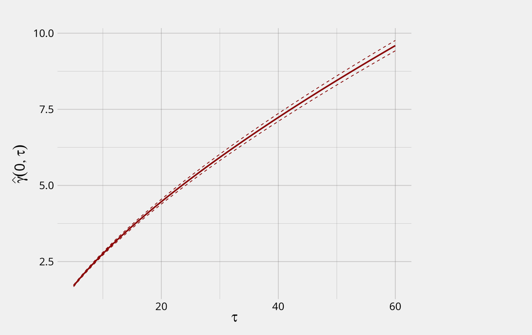
Temporal dependence

Extremogram estimator:

For $s \in \mathcal{S}$, a high quantile q and $t_k \in \{t_1, \ldots, t_T\}$,

$$\widehat{\chi}_q^{(\boldsymbol{s})}(\boldsymbol{0}, \tau) = \frac{\frac{1}{T-\tau}\sum_{k=1}^{T-\tau} \mathbb{1}_{\{X^*_{s,t_k} > q, X^*_{s,t_k+\tau} > q\}}}{\frac{1}{T}\sum_{k=1}^{T} \mathbb{1}_{\{X^*_{s,t_k} > q\}}}$$

Transformation and WLSE ⟶

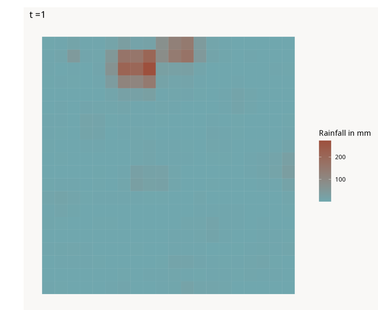


Temporal variogram estimate $\widehat{\gamma}(\boldsymbol{0}, \tau) = 2\widehat{\beta}_2 \tau^{\widehat{\alpha}_2}$

Validation of the separable model

For 100 realisations of a spatio-temporal max-stable Brown-Resnick process

		True	Mean	RMSE	MAE
Spatial	$\widehat{\beta}_1$	0.4	0.445	0.11	0.084
	$\widehat{\alpha}_1$	1.5	1.465	0.159	0.129
Temporal	$\widehat{\beta}_2$	0.2	0.263	0.092	0.075
	$\widehat{\alpha}_2$	1	0.888	0.137	0.118

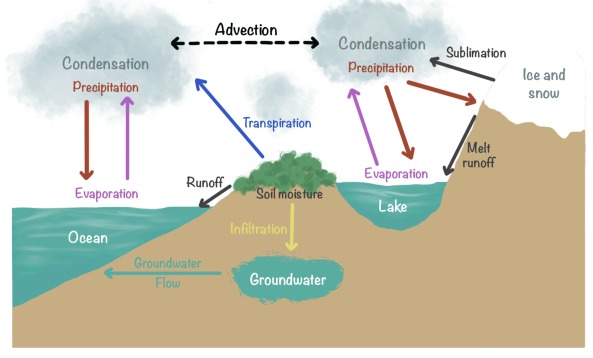


A realisation of a max-stable Brown-Resnick process

Beyond separability: advection

Advection vector $\boldsymbol{V}$

- **What?** Horizontal transport of air masses
- **Why?** To relax the separability assumption
- **In the model?** Lagrangian/Euleurian: $\gamma_L(\boldsymbol{h}, \tau) = \gamma(\boldsymbol{h} - \tau\boldsymbol{V}, \tau)$ Model: $\frac{1}{2}\gamma_L(\boldsymbol{h}, \tau) = \beta_1 \|\boldsymbol{h} - \tau\boldsymbol{V}\|^{\alpha_1} + \beta_2 \tau^{\alpha_2}$



- **Estimation?** Parameter optimization of $\boldsymbol{\Theta} = (\beta_1, \beta_2, \alpha_1, \alpha_2, \boldsymbol{V})$

Excesses: for all spatial pairs $(\boldsymbol{s}_i, \boldsymbol{s}_j)$,

$$k_{ij} = \sum_{t=1}^{n} \mathbb{1}_{\{X_{s_i,t} > q, X_{s_j,t} > q\}} \mid n_j \sim \mathcal{B}(n_j, \chi_{ij,\boldsymbol{\Theta}}), \text{ with } n_j = \sum_{t=1}^{n} \mathbb{1}_{\{X_{s_j,t} > q\}}$$

Composite log-likelihood:

$$l_C(\boldsymbol{\Theta}) \propto \sum_{ij} k_{ij} \log \chi_{ij,\boldsymbol{\Theta}} + (n_j - k_{ij}) \log(1 - \chi_{ij,\boldsymbol{\Theta}})$$

Future work

- Combination of the two datasets: downscaling
- Considering non-constant advection
- More complex variogram with anisotropic structure
- Dry events modeling
- Stochastic generator of precipitation

R package on GitHub: chloesrcb/generain

Website: chloesrcb.github.io

References

[1] Pascal Finaud-Guyot et al. *Rainfall data collected by the HSM urban observatory (OMSEV)*. 2023.

[2] Pierre Tabary et al. "A 10-year (1997–2006) reanalysis of Quantitative Precipitation Estimation over France: methodology and first results". In: *IAHS-AISH* (2012).

[3] Philippe Naveau et al. "Modeling jointly low, moderate, and heavy rainfall intensities without a threshold selection". In: *Water Resources Research* (2016).

[4] Bruce M. Brown and Sidney I. Resnick. "Extreme values of independent stochastic processes". In: *Journal of Applied Probability* (1977).

[5] Sven Buhl et al. "Semiparametric estimation for isotropic max-stable space-time processes". In: *Bernoulli* (2019).

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