Modeling moderate and extreme urban rainfall at high spatio-temporal resolution

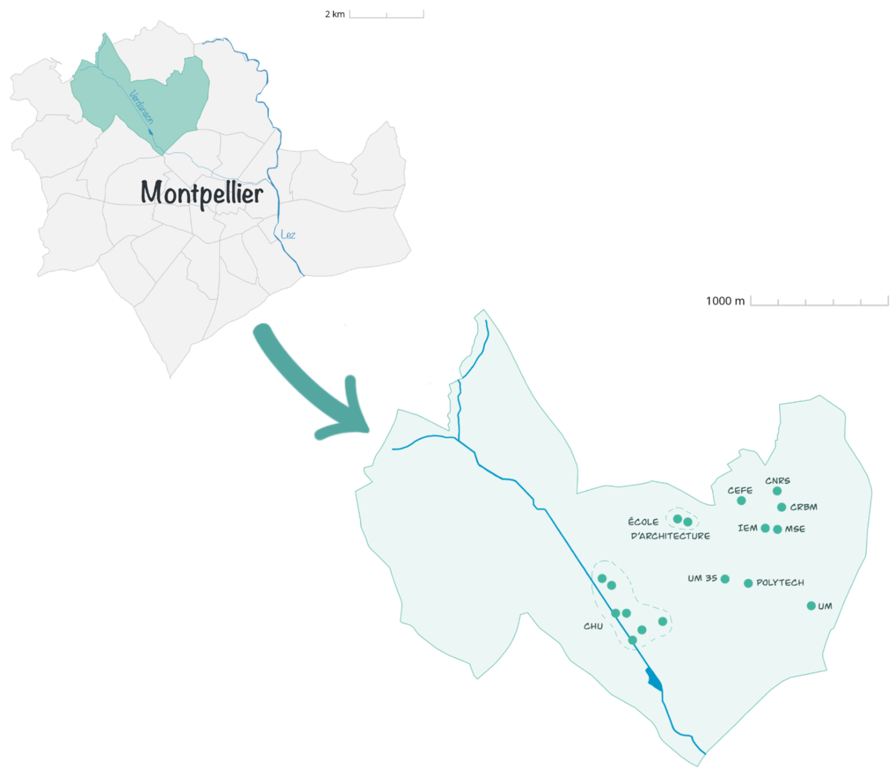
Chloé SERRE-COMBE [1] Nicolas MEYER [1] Thomas OPITZ [2] Gwladys TOULEMONDE [1]

[1]Univ. Montpellier, CNRS, IMAG, Inria, France [2]INRAE, BioSP, Avignon, France

Abstract

- High spatial and temporal resolution
- Univariate modeling of moderate and intense rainfall
- Analysis of the spatio-temporal extremal dependence
- Weighted least squares model for dependence modeling

Data [1]



$\mathcal{S} = \{17 \text{ rain gauges}\}$

- Period: $[2019, 2022]$
- High temporal resolution:
  Every minute ⟶ 5-minute aggregation
- High spatial resolution:
  Interdistance $\in [77, 1531]$ meters

Univariate modeling of the rainfall distribution

Let Y denote the rainfall measurement at a given site.

Generalized Pareto Distribution (GPD) [2] for rainfall excesses above a high threshold u

$$Y - u \mid Y > u \sim H_\xi, \quad \text{where} \quad H_\xi\left(\frac{y}{\sigma}\right) = \begin{cases} 1 - \left(1 + \xi \frac{y}{\sigma}\right)_+^{-1/\xi} & \text{si } \xi \neq 0, \\ 1 - e^{-\frac{y}{\sigma}} & \text{si } \xi = 0, \end{cases}$$

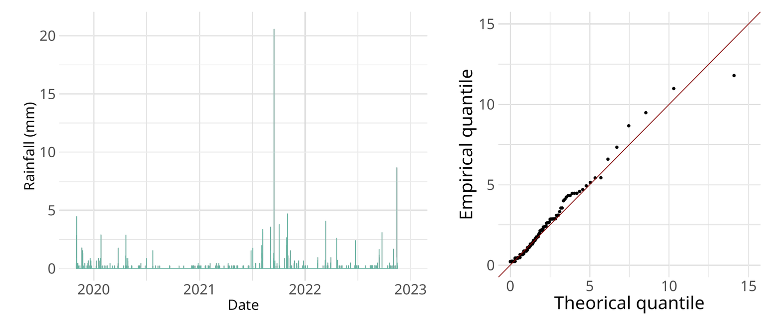
where $a_+ = \max(a, 0)$, $\sigma > 0$, $\xi \in \mathbb{R}$ and $y > 0$.

Extended Generalized Pareto Distribution (EGPD) [3] for the entire distribution

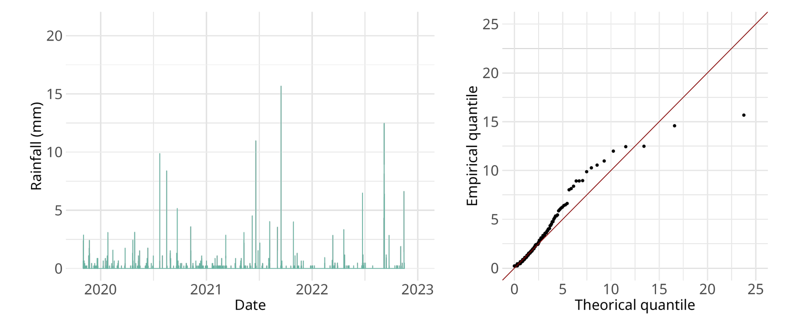
The cumulative distribution function (cdf) of the EGPD is given by

$$F_Y(y) = G\left(H_\xi\left(\frac{y}{\sigma}\right)\right),$$

with $G(x) = x^\kappa$, $\kappa > 0$.



CNRS



Polytech

EGPD fit with $\widehat{\kappa} = 0.56$, $\widehat{\sigma} = 0.26$ and $\widehat{\xi} = 0.51$

Spatio-temporal dependence modeling [4]

Let $X = \{X(s,t),\ (s,t) \in \mathcal{S} \times [0, \infty)\}$ be a strictly stationary isotropic Brown-Resnick process.
For a spatial lag $v \geq 0$ and a temporal lag $h \geq 0$, the extremogram of X is given by

$$\chi(v,h) = 2\left(1 - \phi\left(\sqrt{\frac{1}{2}\delta(v,h)}\right)\right)$$

with ϕ the standard normal distribution function and δ a stationary and isotropic variogram.

Assumption of additive separability: $\frac{\delta(v,h)}{2} = \theta_1 v^{\alpha_1} + \theta_2 h^{\alpha_2}$, $0 < \alpha_1, \alpha_2 \leq 2$, $\theta_1, \theta_2 > 0$

Spatio-temporal
$\chi(v,h) = 2\left(1 - \phi\left(\sqrt{\frac{1}{2}\delta(v,h)}\right)\right)$

Transformation:
$\eta(\chi) = 2\log\left(\phi^{-1}\left(1 - \frac{1}{2}\chi\right)\right)$

Spatial
$\eta(\chi(v,0)) = \log\theta_1 + \alpha_1 \log v,\ v > 0$
$:= c_1 + \alpha_1 x_v$

Temporal
$\eta(\chi(0,h)) = \log\theta_2 + \alpha_2 \log h,\ h > 0$
$:= c_2 + \alpha_2 x_h$

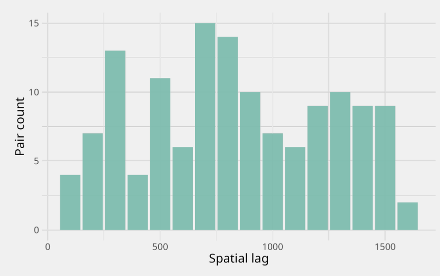
Weighted Least Squares Estimation (WLSE)

$$\begin{pmatrix}\widehat{c}_i \\ \widehat{\alpha}_i\end{pmatrix} = \operatorname{argmin}_{c_i, \alpha_i} \sum_x w_x \left(\eta(\widehat{\chi}) - (c_i + \alpha_i x)\right)^2$$

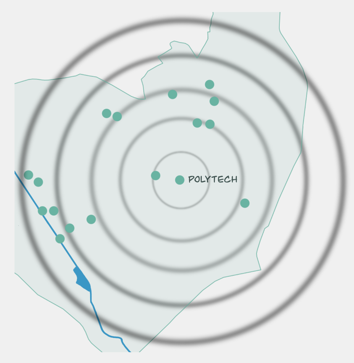
Spatial extremogram

For all spatial lags $v = k \times \Delta v$, $k = 1, 2, \ldots$, we define

$$N(v) = \left\{(s_i, s_j) \mid \|s_i - s_j\| \in]v - \Delta v, v]\right\}$$



For $\Delta v = 100$ meters



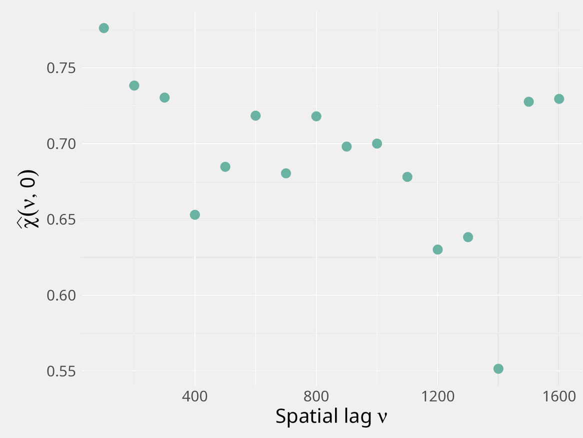
For fixed t and for any $(s_i, s_j) \in N(v)$

$$\chi_{ij,q}^{(t)}(v,0) = \frac{\mathbb{P}\left(X(s_i,t) > q, X(s_j,t) > q\right)}{\mathbb{P}\left(X(s_i,t) > q\right)}$$

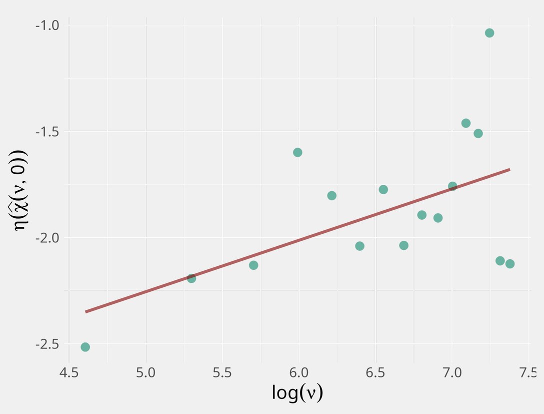
Estimator:

$$\widehat{\chi}_q^{(t)}(v,0) = \frac{\frac{1}{|N(v)|}\sum_{i,j \mid (s_i,s_j) \in N(v)} \mathbb{1}_{\{X(s_i,t)>q, X(s_j,t)>q\}}}{\frac{1}{|\mathcal{S}|}\sum_{i=1}^{|\mathcal{S}|} \mathbb{1}_{\{X(s_i,t)>q\}}}$$

with q a high quantile (99.8%).



WLSE



	Estimate	Std. Error
$\widehat{c}_1$	-3.465^{***}	0.605
$\widehat{\alpha}_1$	0.242^{*}	0.093

*p-value< 0.05; ***p-value< 0.001

Temporal extremogram

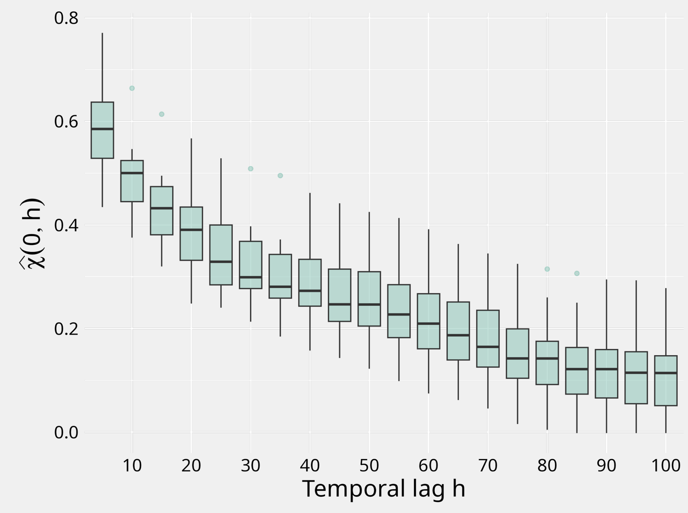
For a site s and for any t

$$\chi_{t,q}^{(s)}(0,h) = \frac{\mathbb{P}\left(X(s,t) > q, X(s,t+h) > q\right)}{\mathbb{P}\left(X(s,t) > q\right)}$$

Estimator:

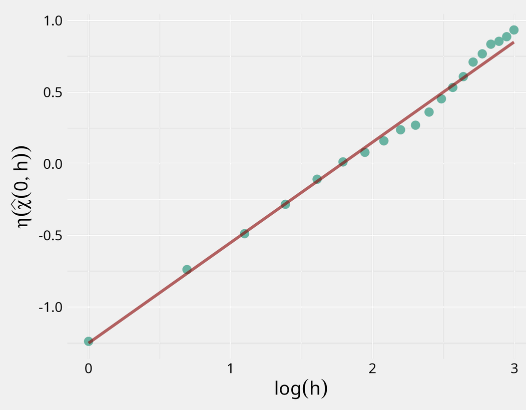
$$\widehat{\chi}_q^{(s)}(0,h) = \frac{\frac{1}{T-h}\sum_{k=1}^{T-h} \mathbb{1}_{\{X(s,t_k)>q, X(s,t_k+h)>q\}}}{\frac{1}{T}\sum_{k=1}^{T} \mathbb{1}_{\{X(s,t_k)>q\}}}$$

with q a high quantile (99.8%) and $t_k \in \{t_1, \ldots, t_T\}$.



With $\Delta h = 5$ minutes

WLSE

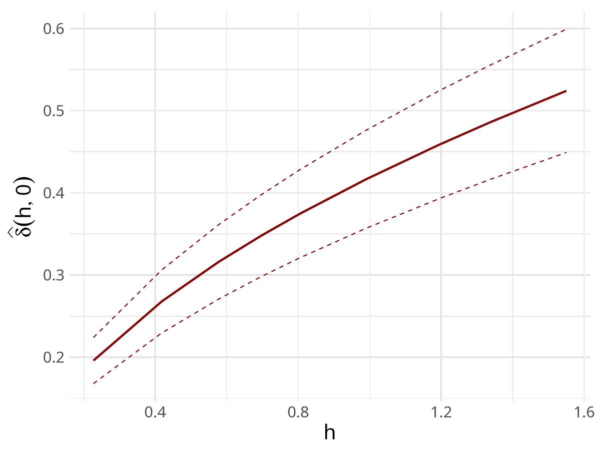


	Estimate	Std. Error
$\widehat{c}_2$	-1.252^{***}	0.023
$\widehat{\alpha}_2$	0.702^{***}	0.012

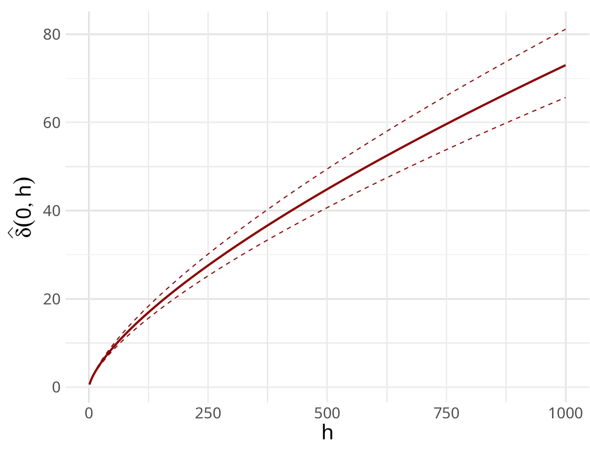
***p-value< 0.001

Empirical variogram

Spatial
$\widehat{\delta}(v,0) = 2\widehat{\theta}_1 v^{\widehat{\alpha}_1}$



Temporal
$\widehat{\delta}(0,h) = 2\widehat{\theta}_2 h^{\widehat{\alpha}_2}$



Future works

- More complex variograms, including space-time non-separability
- Anisotropic structure and advection
- Combination of small-scale and larger-scale spatio-temporal modeling using different data sources

References

[1] Pascal Finaud-Guyot et al. *Rainfall data collected by the HSM urban observatory (OMSEV)*. 2023.
[2] Stuart Coles et al. *An introduction to statistical modeling of extreme values*. Vol. 208. Springer, 2001.
[3] Philippe Naveau et al. "Modeling jointly low, moderate, and heavy rainfall intensities without a threshold selection". In: *Water Resources Research* (2016).
[4] Sven Buhl et al. "Semiparametric estimation for isotropic max-stable space-time processes". In: *Bernoulli* 25.4A (2019), pp. 2508–2537.

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