Moderate and extreme urban rainfall modeling at a fine spatio-temporal resolution

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Chloé SERRE-COMBE [1] Nicolas MEYER [1] Thomas OPITZ [2] Gwladys TOULEMONDE [1]

[1] *IMAG, Université de Montpellier, LEMON Inria*

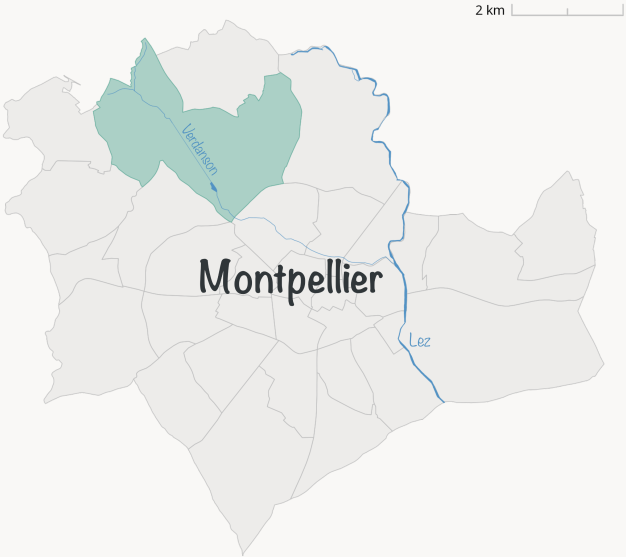
[2] *INRAE, BioSP, Avignon*

General context

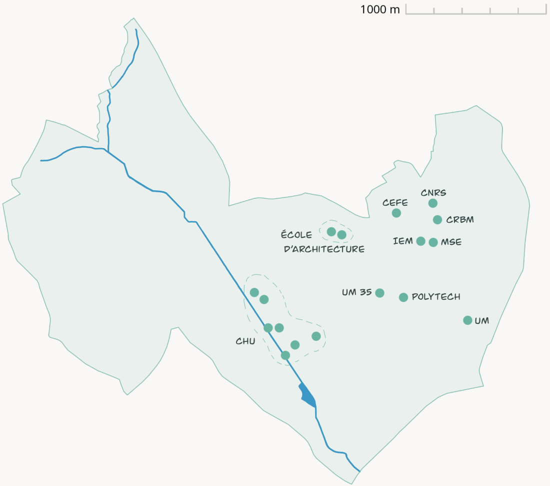
Floods in Montpellier, September 2022 and August 2015 (*Midi Libre*)

- Mediterranean events, localized rainfall
- Urban area, flood risks

Study area



- North of Montpellier, *Hôpitaux-Facultés* district
- Verdanson water catchment, tributary of the Lez river



- **Source:** Urban observatory of HydroScience Montpellier (OHSM)[1]
- **Time period:** [2019, 2023]
- **Temp. resol.:** 5 minutes
- **Spatial resol.:** 77m to 1531m

[1] Finaud-Guyot et al. 2023

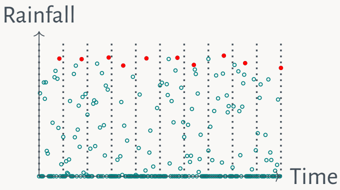
$\mathcal{S}$ = {**17 rain gauges**} $\subset \mathbb{R}^2$ and $\mathcal{T} \subset \mathbb{R}_+$

Univariate precipitation modeling

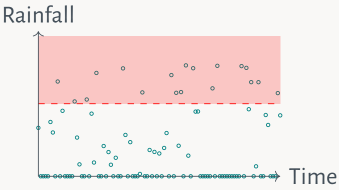
Extreme precipitation

Block-maxima

Peaks Over Threshold



Generalized Extreme Value distribution (GEV)



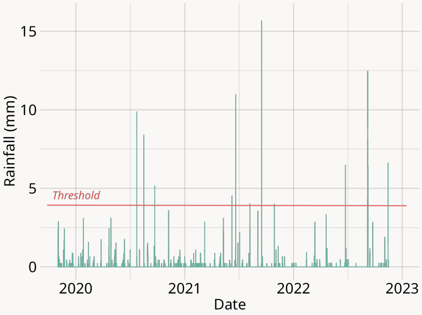
Generalized Pareto Distribution (GPD)

Generalized Pareto Distribution

$$\overline{H}_\xi\left(\frac{x-u}{\sigma}\right) = \begin{cases} \left(1+\xi\frac{x-u}{\sigma}\right)_+^{-1/\xi} & \text{if } \xi \neq 0\,, \\ e^{-\frac{x-u}{\sigma}} & \text{if } \xi = 0\,, \end{cases}$$

where $a_+ = \max(a, 0)$, $\sigma > 0$, $x - u > 0$

- Models extreme precipitation
- Depends on a threshold choice



Generalized Pareto Distribution → **Extended GPD**[2]

$$\overline{H}_\xi\left(\frac{x-u}{\sigma}\right) = \begin{cases} \left(1+\xi\frac{x-u}{\sigma}\right)_+^{-1/\xi} & \text{if } \xi \neq 0\,, \\ e^{-\frac{x-u}{\sigma}} & \text{if } \xi = 0\,, \end{cases}$$

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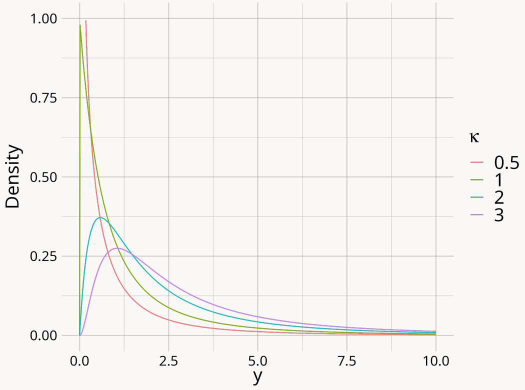
- Models extreme precipitation
- Depends on a threshold choice

$$F(x) = G\left(H_\xi\left(\frac{x}{\sigma}\right)\right),$$

where $G(x) = x^\kappa$, $\kappa > 0$

- Models **moderate and extreme** precipitation
- Avoids a threshold choice

[2] NAVEAU et al. 2016



$\sigma = 1, \ \xi = 0.5$

Extended GPD

$$F(x) = G\left(H_\xi\left(\frac{x}{\sigma}\right)\right),$$

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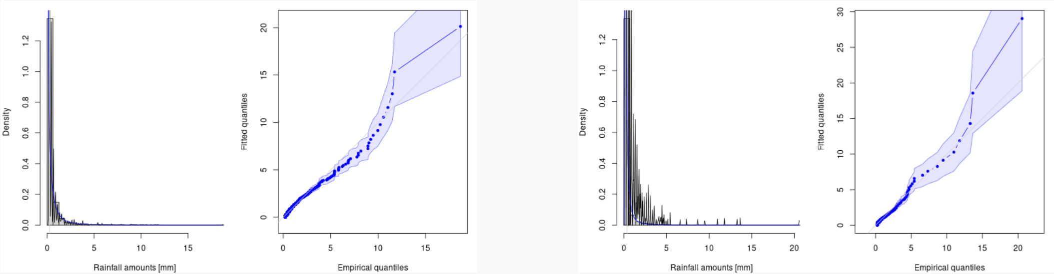


EGPD fitting for two rain gauges, CRBM (left) and CNRS (right) with left-censoring and 95% CI

Rainfall field: $\boldsymbol{X} = \{X_{\boldsymbol{s},t}\,,\ (\boldsymbol{s}, t) \in \mathcal{S} \times \mathcal{T}\}$

Domain of attraction: Stationary isotropic max-stable Brown-Resnick process

Brown-Resnick process (Brown and Resnick 1977)

For all $\boldsymbol{s} \in \mathcal{S}$ and $t \in \mathcal{T}$,

$$X_{\boldsymbol{s},t} = \bigvee_{j=1}^{\infty} \xi_j e^{W^j_{\boldsymbol{s},t} - \gamma(\boldsymbol{s},t)}$$

- ξ_j: point of a Poisson process with intensity $\xi^{-2} d\xi$
- W^j: independent replicates of an intrinsic stationary and isotropic Gaussian random field $\boldsymbol{W}$
- γ: spatio-temporal variogram of $\boldsymbol{W}$

Dependence measures

Let $\Lambda_{\mathcal{S}} \subset \mathbb{R}_+^2$ and $\Lambda_{\mathcal{T}} \subset \mathbb{R}_+$ be sets of spatial and temporal lags respectively.

Spatio-temporal extremogram

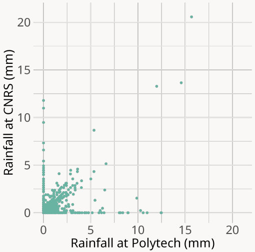
For all $\boldsymbol{h} \in \Lambda_{\mathcal{S}}, \tau \in \Lambda_{\mathcal{T}}$,

$$\chi(\boldsymbol{h}, \tau) = \lim_{q \to 1} \mathbb{P}(X^*_{\boldsymbol{s},t} > q \mid X^*_{\boldsymbol{s}+\boldsymbol{h},t+\tau} > q),$$

with $q \in [0, 1[$ and $X^*_{\boldsymbol{s},t}$ the uniform margins.

Spatio-temporal variogram γ

$$\gamma(\boldsymbol{h}, \tau) = \frac{1}{2} Var \left(W_{\boldsymbol{s},t} - W_{\boldsymbol{s}+\boldsymbol{h},t+\tau} \right), \boldsymbol{h} \in \Lambda_{\mathcal{S}}, \tau \in \Lambda_{\mathcal{T}}$$



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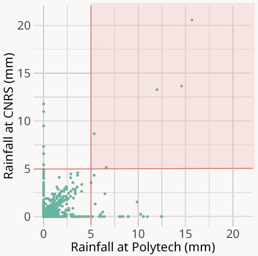


Spatio-temporal extremogram of a Brown-Resnick process

Let $\boldsymbol{h} \in \Lambda_{\mathcal{S}}$ and $\tau \in \Lambda_{\mathcal{T}}$. We have

$$\chi(\boldsymbol{h}, \tau) = 2\left(1 - \phi\left(\sqrt{\frac{1}{2}\gamma(\boldsymbol{h}, \tau)}\right)\right)$$

with ϕ the std normal c.d.f. and γ the variogram of $\boldsymbol{W}$.

Dependence model framework: BUHL et al. 2019

Case of additive separability: $\frac{\gamma(\boldsymbol{h},\tau)}{2} = \beta_1 ||\boldsymbol{h}||^{\alpha_1} + \beta_2 \tau^{\alpha_2}, \ 0 < \alpha_1, \alpha_2 \leq 2, \ \beta_1, \beta_2 > 0$

Spatio-temporal

$$\chi(\boldsymbol{h},\tau) = 2\left(1 - \phi\left(\sqrt{\tfrac{1}{2}\gamma(\boldsymbol{h},\tau)}\right)\right)$$

Transformation:

$$\eta(\chi) = 2\log\left(\phi^{-1}\left(1 - \tfrac{1}{2}\chi\right)\right)$$

Spatial

$$\eta\left(\chi(\boldsymbol{h},0)\right) = \log\beta_1 + \alpha_1 \log||\boldsymbol{h}||\,, \ \ \boldsymbol{h} \in \Lambda_{\mathcal{S}}$$

Temporal

$$\eta\left(\chi(0,\tau)\right) = \log\beta_2 + \alpha_2 \log\tau\,, \ \ \tau \in \Lambda_{\mathcal{T}}$$

Spatio-temporal dependence modeling

Case of additive separability: $\frac{\gamma(\boldsymbol{h},\tau)}{2} = \beta_1 ||\boldsymbol{h}||^{\alpha_1} + \beta_2 \tau^{\alpha_2}, \; 0 < \alpha_1, \alpha_2 \leq 2\,, \; \beta_1, \beta_2 > 0$

Spatio-temporal

$$\chi(\boldsymbol{h},\tau) = 2\left(1 - \phi\left(\sqrt{\tfrac{1}{2}\gamma(\boldsymbol{h},\tau)}\right)\right)$$

Transformation:

$$\eta(\chi) = 2\log\left(\phi^{-1}\left(1 - \tfrac{1}{2}\chi\right)\right)$$

Spatial

$$\eta\left(\chi(\boldsymbol{h},0)\right) := c_1 + \alpha_1 x_h\,, \;\; \boldsymbol{h} \in \Lambda_{\mathcal{S}}$$

Temporal

$$\eta\left(\chi(0,\tau)\right) := c_2 + \alpha_2 x_\tau\,, \;\; \tau \in \Lambda_{\mathcal{T}}$$

Weighted Least Squares Estimation (WLSE)

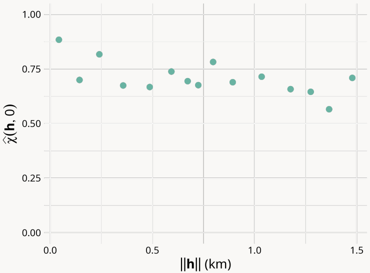
$$\begin{pmatrix} \widehat{c}_i \\ \widehat{\alpha}_i \end{pmatrix} = \operatorname{argmin}_{c_i,\alpha_i} \sum_x w_x \left(\eta\left(\widehat{\chi}\right) - (c_i + \alpha_i x)\right)^2$$

Empirical spatial extremogram

For a fixed $t \in \mathcal{T}$ and q a high quantile,

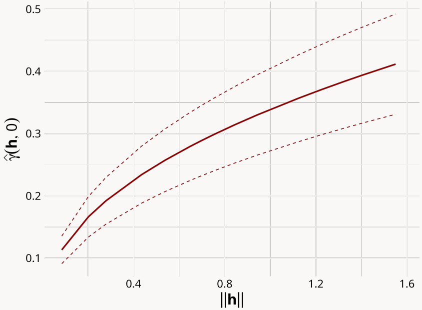
$$\widehat{\chi}_q^{(t)}(\boldsymbol{h}, 0) = \frac{\frac{1}{|N_{\boldsymbol{h}}|} \sum_{i,j \,|\, (\boldsymbol{s}_i, \boldsymbol{s}_j) \in N_{\boldsymbol{h}}} \mathbb{1}_{\{X^*_{\boldsymbol{s}_i,t} > q \,, X^*_{\boldsymbol{s}_j,t} > q\}}}{\frac{1}{|\mathcal{S}|} \sum_{i=1}^{|\mathcal{S}|} \mathbb{1}_{\{X^*_{\boldsymbol{s}_i,t} > q\}}},$$

where $C_{\boldsymbol{h}}$ are equifrequent distance classes and $N_{\boldsymbol{h}} = \left\{(\boldsymbol{s}_i, \boldsymbol{s}_j) \in \mathcal{S}^2 \;\middle|\; \|\boldsymbol{s}_i - \boldsymbol{s}_j\| \in C_{\boldsymbol{h}}\right\}$.



Transformation and WLSE

$\Longrightarrow$

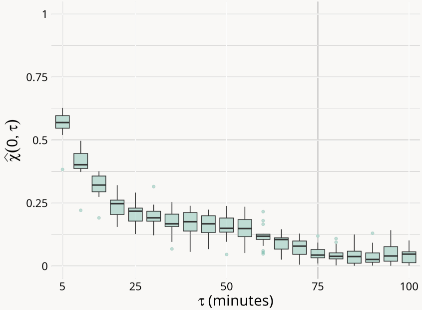


Spatial variogram $\widehat{\gamma}(\boldsymbol{h}, 0) = 2\widehat{\beta}_1 \|\boldsymbol{h}\|^{\widehat{\alpha}_1}$

Empirical temporal extremogram

For a location $s \in \mathcal{S}$, a high quantile q and $t_k \in \{t_1, \ldots, t_T\}$,

$$\widehat{\chi}_q^{(\boldsymbol{s})}(\mathbf{0}, \tau) = \frac{\frac{1}{T-\tau}\sum_{k=1}^{T-\tau} \mathbb{1}_{\{X^*_{\boldsymbol{s},t_k} > q\, , X^*_{\boldsymbol{s},t_k+\tau} > q\}}}{\frac{1}{T}\sum_{k=1}^{T} \mathbb{1}_{\{X^*_{\boldsymbol{s},t_k} > q\}}}$$



Transformation and WLSE

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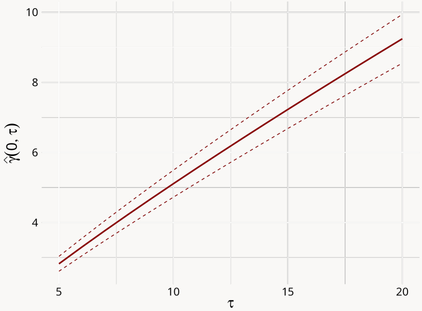


Temporal variogram $\widehat{\gamma}(\mathbf{0}, \tau) = 2\widehat{\beta}_2 \tau^{\widehat{\alpha}_2}$

Advection vector $\boldsymbol{V}$

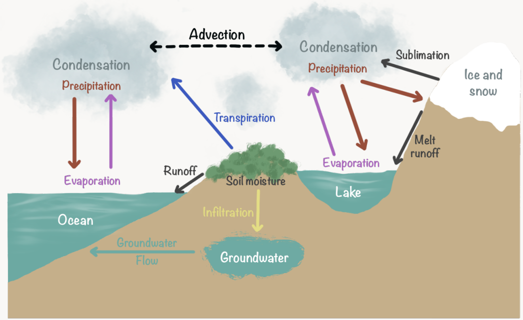
- Horizontal transport of air masses
- To relax the separability assumption

Lagrangian/Eulerian variogram

$$\gamma_L(\boldsymbol{h}, \tau) = \gamma(\boldsymbol{h} - \tau\boldsymbol{V}, \tau)$$

Dependence model

$$\frac{1}{2}\gamma_L(\boldsymbol{h}, \tau) = \beta_1 \|\boldsymbol{h} - \tau\boldsymbol{V}\|^{\alpha_1} + \beta_2 \tau^{\alpha_2}$$



Hydrologic cycle

Parameter optimization of $\boldsymbol{\Theta} = (\beta_1, \beta_2, \alpha_1, \alpha_2, \boldsymbol{V})$

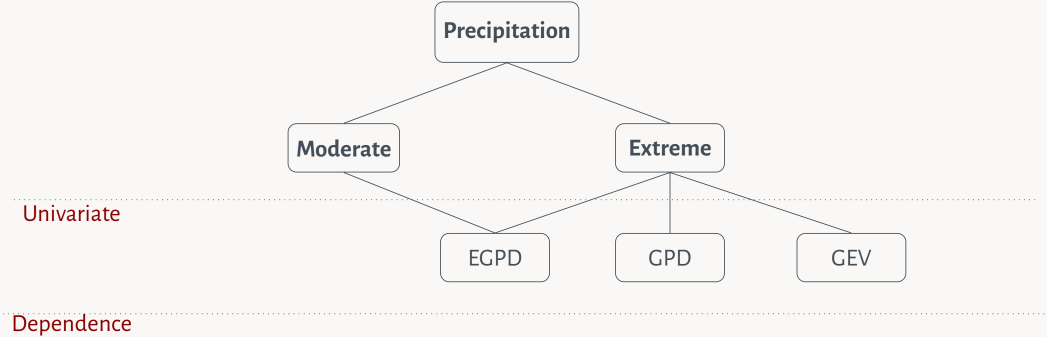
Excesses: for all spatial pairs $(\boldsymbol{s}_i, \boldsymbol{s}_j)$,

$$k_{ij} = \sum_{t=1}^{n} \mathbb{1}_{\{X_{\boldsymbol{s}_i,t}>q, X_{\boldsymbol{s}_j,t}>q\}} \mid n_j \sim \mathcal{B}(n_j, \chi_{ij,\boldsymbol{\Theta}})$$

Composite log-likelihood:

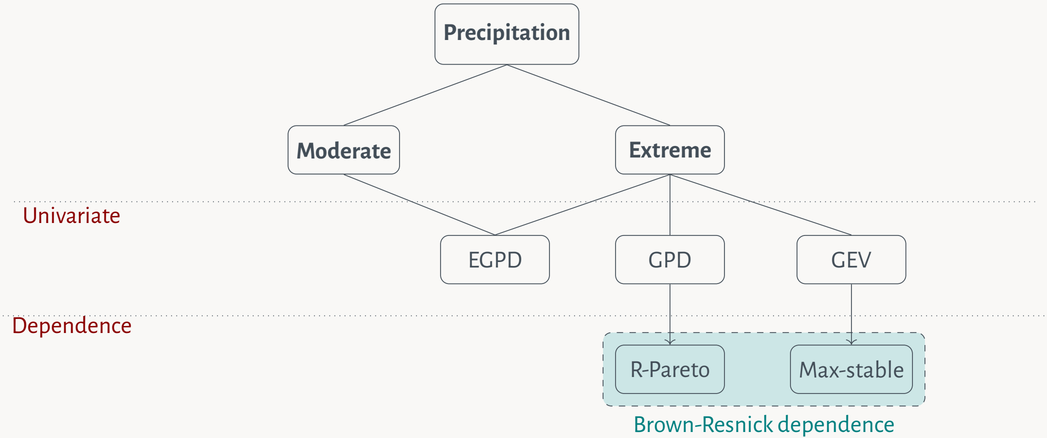
$$l_C(\boldsymbol{\Theta}) \propto \sum_{ij} k_{ij} \log \chi_{ij,\boldsymbol{\Theta}} + (n_j - k_{ij}) \log(1 - \chi_{ij,\boldsymbol{\Theta}})$$

Conclusion



Precipitation
Moderate
Extreme
Univariate
EGPD
GPD
GEV
Dependence

Conclusion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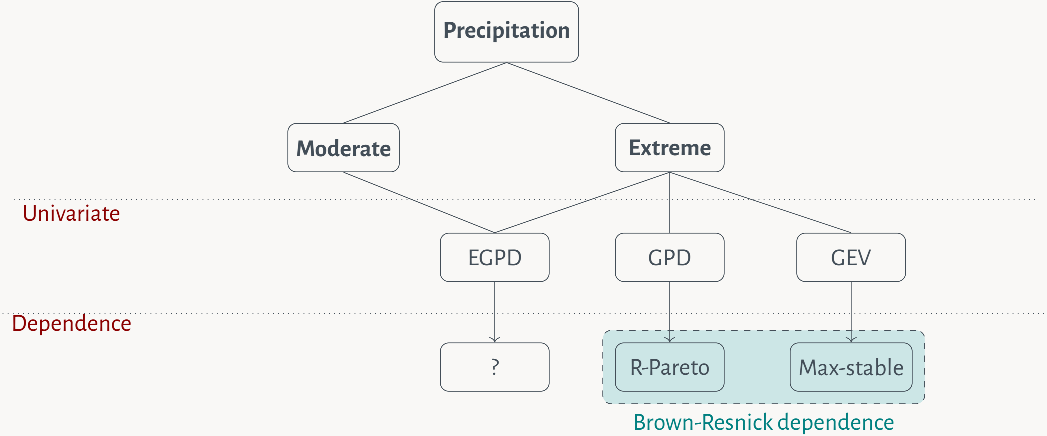

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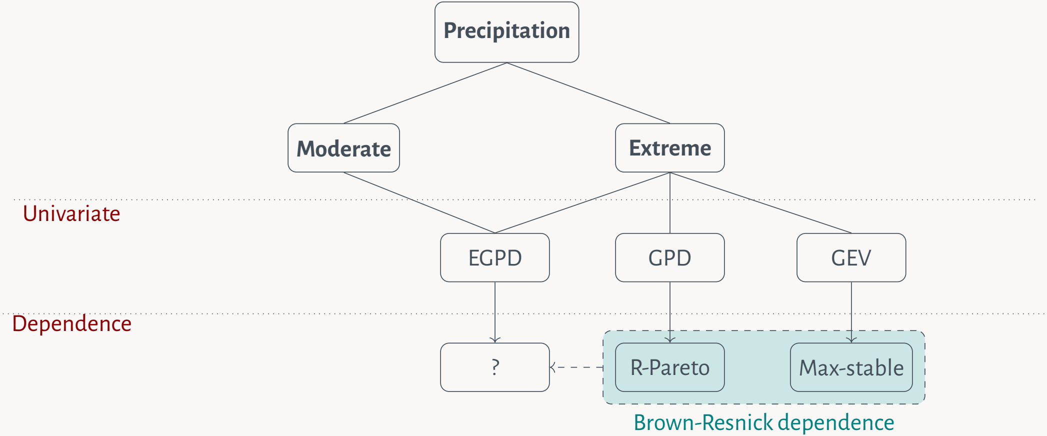

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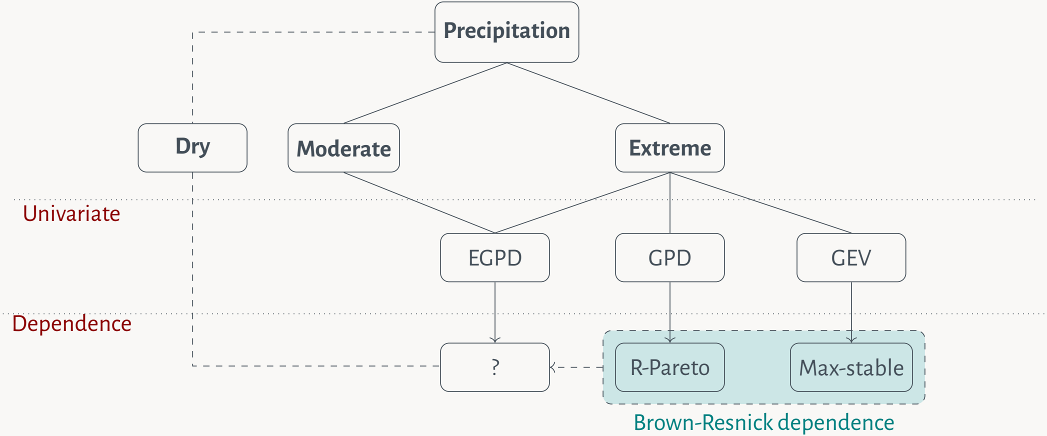

Precipitation
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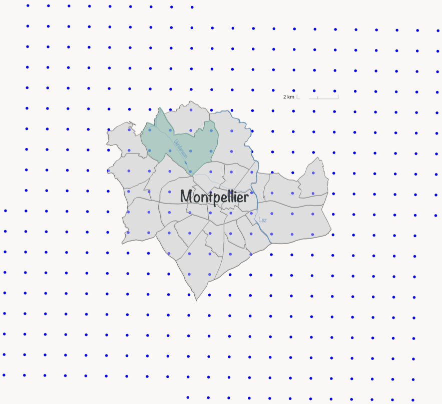

- **Source:** COMEPHORE, Météo France
- **Time period:** [1997, 2023]
- **Temporal resolution:** Every hour
- **Spatial resolution:** 1 km^2

$\mathcal{S} = \{\textbf{400 pixels}\} \subset \mathbb{R}^2$ and $\mathcal{T} \subset \mathbb{R}_+$

References

Brown, Bruce M. and Sidney I. Resnick (1977). “Extreme values of independent stochastic processes”. In: *Journal of Applied Probability*. DOI: 10.2307/3213346.

Buhl, Sven et al. (2019). “Semiparametric estimation for isotropic max-stable space-time processes”. In: *Bernoulli*.

Finaud-Guyot, Pascal et al. (2023). *Rainfall data collected by the HSM urban observatory (OMSEV)*. DOI: 10.23708/67LC36.

Haruna, Abubakar, Juliette Blanchet, and Anne-Catherine Favre (2023). *Modeling Areal Precipitation Hazard: A Data-driven Approach to Model Intensity-Duration-Area-Frequency Relationships using the Full Range of Non-Zero Precipitation in Switzerland*. preprint. Preprints. DOI: 10.22541/essoar.169111775.53035997/v1.

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Model validation - simulations

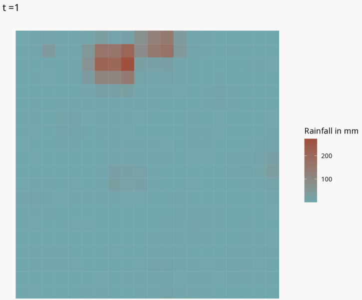
Brown-Resnick simulations

- Spatial: $\mathcal{S} = \{\,400 \text{ sites}\,\}$, $\mathcal{T} = \{1, \ldots, 50\}$, $|\Lambda_{\mathcal{S}}| = 10$ and $|\Lambda_{\mathcal{T}}| = 10$
- Temporal: $\mathcal{S} = \{\,25 \text{ sites}\,\}$, $\mathcal{T} = \{1, \ldots, 300\}$, $|\Lambda_{\mathcal{S}}| = 10$ and $|\Lambda_{\mathcal{T}}| = 10$

	True	Mean	RMSE	MAE
$\widehat{\beta}_1$	0.4	0.524	0.138	0.126
$\widehat{\alpha}_1$	1.5	1.507	0.120	0.088
$\widehat{\beta}_2$	0.2	0.259	0.093	0.074
$\widehat{\alpha}_2$	1	0.873	0.149	0.128

Parameter estimates for 100 realisations

True variogram: $\frac{1}{2}\gamma\left(\boldsymbol{h}, \tau\right) = 0.4\|\boldsymbol{h}\|^{3/2} + 0.2\tau$



Advection vector $\boldsymbol{V}$

- Horizontal transport of air masses
- To relax the separability assumption

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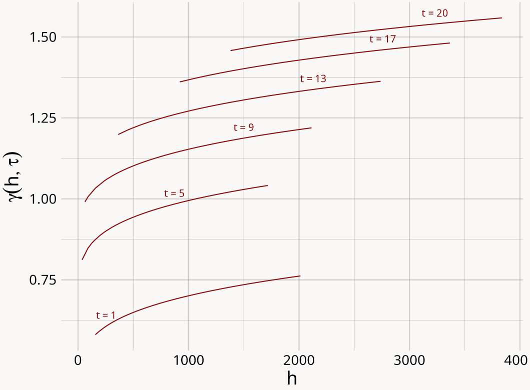
Dependence model

$$\frac{1}{2}\gamma_L(\boldsymbol{h}, \tau) = \beta_1 \|\boldsymbol{h} - \tau\boldsymbol{V}\|^{\alpha_1} + \beta_2 \tau^{\alpha_2}$$



Spatial variogram with a constant advection $\boldsymbol{V} = (0.001, 45)^T$ on OHSM data

Beyond separability: advection

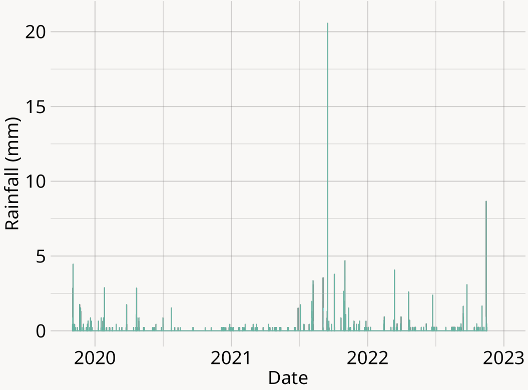
- A relation between time and space via an advection vector $\boldsymbol{V}$:
  $\hookrightarrow$ it models the shift of rain storms above the ground
- Impact on the variogram: $\gamma_V(h,\tau) = \gamma(h - \tau \boldsymbol{V}, \tau)$
- For our model:

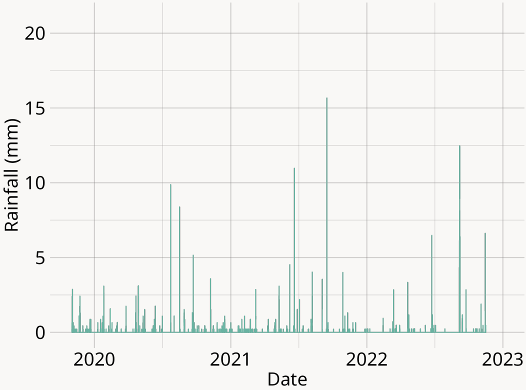
$$\frac{1}{2}\gamma_V(h,\tau) = \beta_1 \|h - \tau \boldsymbol{V}\|^{\alpha_1} + \beta_2 \tau^{\alpha_2}$$

	OHSM without $\boldsymbol{V}$	COMPEHORE without $\boldsymbol{V}$	COMPEHORE with $\boldsymbol{V}$
$\widehat{\beta}_1$	0.024	0.006	0.006
$\widehat{\alpha}_1$	0.286	1.588	1.603
$\widehat{\beta}_2$	0.091	0.419	0.480
$\widehat{\alpha}_2$	0.696	0.892	0.731
V_x	-	-	-0.00027
V_y	-	-	0.00089

Parameter estimates with and without advection

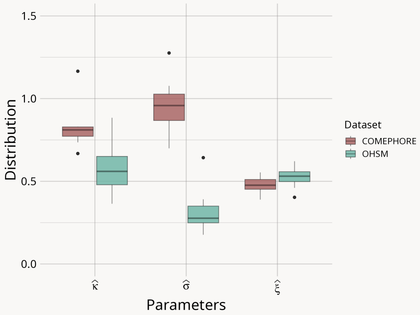
Rainfall data - OHSM



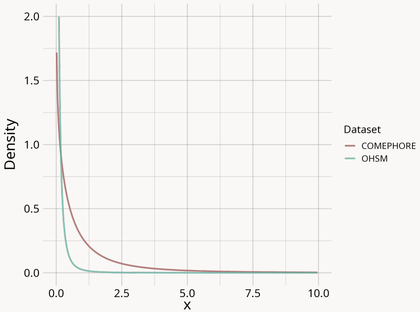


Rainfall amounts on CNRS and Polytech rain gauges

Left-censoring: selected according to the NRMSE criterion for each site individually[3]



Parameter estimates



Density with mean parameters

[3] HARUNA, BLANCHET, and FAVRE 2023

r-Pareto process

- Extension of the GPD approach for spatio-temporal processes
- In our case, a r-Pareto process is given by

$$X_{s,t} = R^{W - W(s_0,t_0) - \gamma(s,t,s_0,t_0)},$$

  with (s_0, t_0) a given space-time location.
- Very similar to the previous process but here a specific space-time location is fixed