

MODERATE AND EXTREME URBAN RAINFALL MODELING AT A FINE SPATIO-TEMPORAL RESOLUTION

Journées MIPS, Montpellier, June 2024

Chloé SERRE-COMBE¹ Nicolas MEYER¹ Thomas OPITZ² Gwladys TOULEMONDE¹

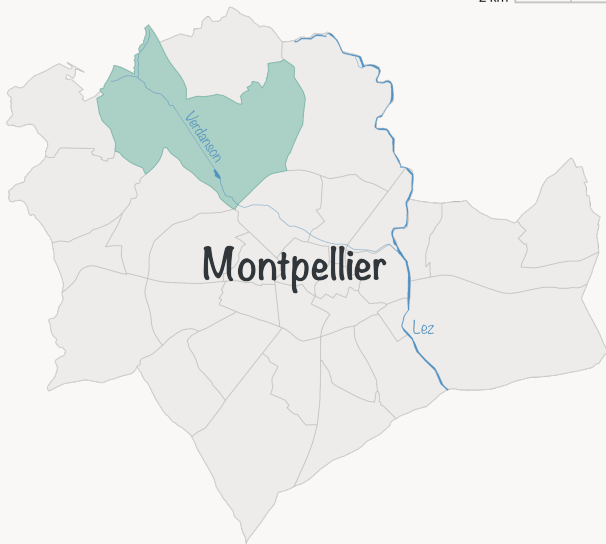
¹IMAG, Université de Montpellier, LEMON Inria

²INRAE, BioSP, Avignon



STUDY AREA

2 km



- **Geography:**
Verdanson water catchment, tributary of the Lez, located in an urban area
- **Context:**
Mediterranean events, flood risks

RAIN GAUGES NETWORK



- **Source:** Urban observatory of HydroScience Montpellier (OHSM)¹
- **Time period:** [2019, 2022]
- **High temporal resolution:**
Every minute → 5-minute aggregation
- **High spatial resolution:**
Interdistance $\in [77, 1531]$ meters

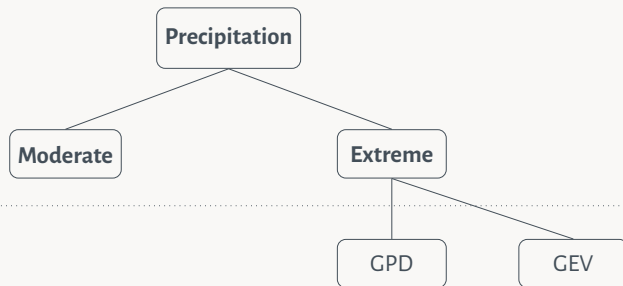
¹FINAUD-GUYOT et al. 2023

$$\mathcal{S} = \{17 \text{ rain gauges}\} \subset \mathbb{R}^2 \text{ and } \mathcal{T} \subset \mathbb{R}_+$$

PRECIPITATION MODELING

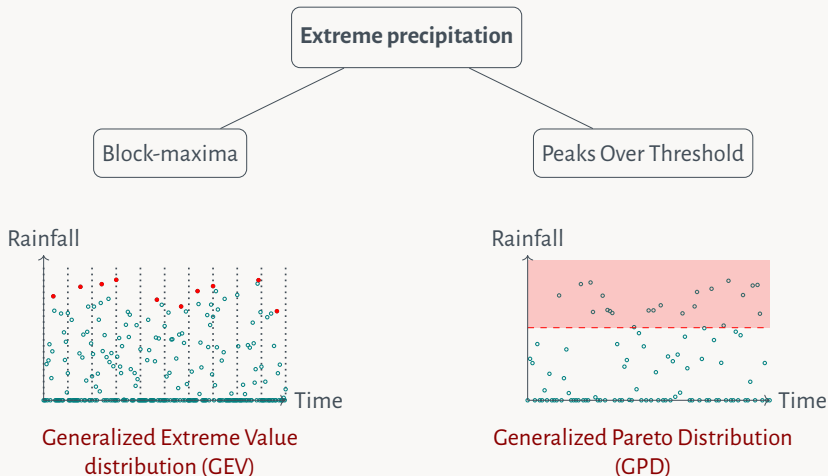


PRECIPITATION MODELING



Univariate

UNIVARIATE PRECIPITATION MODELING

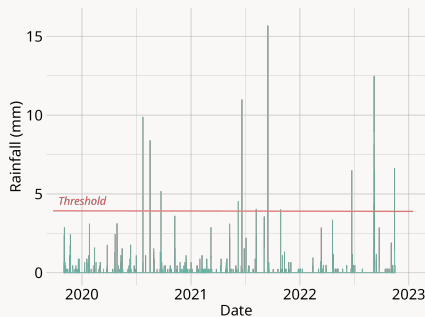


Generalized Pareto Distribution

$$\bar{H}_\xi \left(\frac{x-u}{\sigma} \right) = \begin{cases} (1 + \xi \frac{x-u}{\sigma})_+^{-1/\xi} & \text{if } \xi \neq 0, \\ e^{-\frac{x-u}{\sigma}} & \text{if } \xi = 0, \end{cases}$$

where $a_+ = \max(a, 0)$, $\sigma > 0$, $x - u > 0$

- Models extreme precipitation
- Depends on a threshold choice



Generalized Pareto Distribution



Extended GPD²

$$\bar{H}_{\xi} \left(\frac{x-u}{\sigma} \right) = \begin{cases} (1 + \xi \frac{x-u}{\sigma})_+^{-1/\xi} & \text{if } \xi \neq 0, \\ e^{-\frac{x-u}{\sigma}} & \text{if } \xi = 0, \end{cases}$$

where $a_+ = \max(a, 0)$, $\sigma > 0$, $x - u > 0$

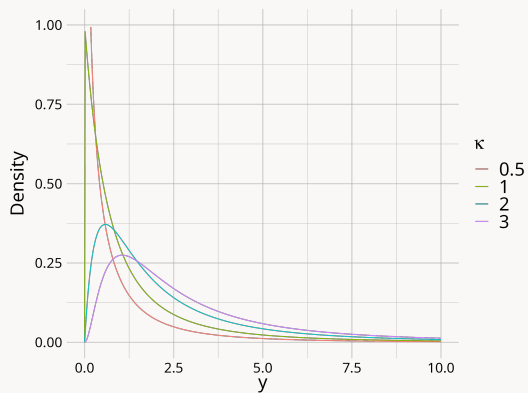
- Models extreme precipitation
- Depends on a threshold choice

$$F(x) = G \left(H_{\xi} \left(\frac{x}{\sigma} \right) \right),$$

where $G(x) = x^{\kappa}$, $\kappa > 0$

- Models **moderate and extreme** precipitation
- Avoids a threshold choice

²NAVEAU et al. 2016



$\sigma = 1, \xi = 0.5$

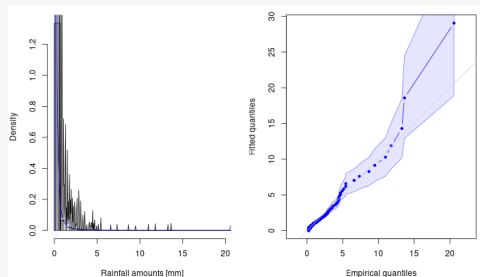
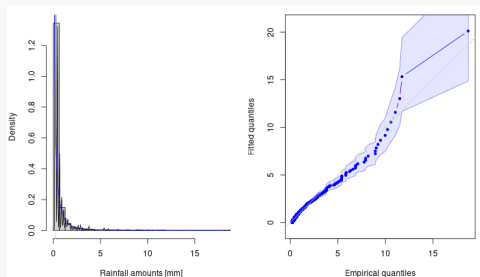
Extended GPD

$$F(x) = G \left(H_{\xi} \left(\frac{x}{\sigma} \right) \right),$$

where $G(x) = x^{\kappa}, \kappa > 0$

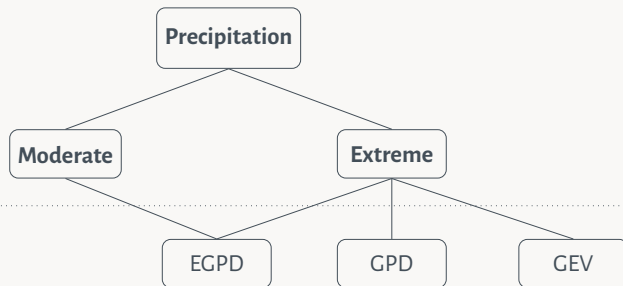
- Models **moderate and extreme** precipitation
- Avoids a threshold choice

EGPD FITTING



EGPD fitting for two rain gauges, CRBM (left) and CNRS (right) with left-censoring and 95% CI

PRECIPITATION MODELING



Univariate

Dependence

Rainfall field: $\mathbf{X} = \{X_{\mathbf{s},t}, (\mathbf{s}, t) \in \mathcal{S} \times \mathcal{T}\}$

Domain of attraction: Stationary isotropic max-stable Brown-Resnick process

Brown-Resnick process (BROWN and RESNICK 1977)

For all $\mathbf{s} \in \mathcal{S}$ and $t \in \mathcal{T}$,

$$X_{\mathbf{s},t} = \bigvee_{j=1}^{\infty} \xi_j e^{W_{\mathbf{s},t}^j - \gamma(\mathbf{s},t)}$$

- ▶ ξ_j : point of a Poisson process with intensity $\xi^{-2} d\xi$
- ▶ W^j : independent replicates of an intrinsic stationary and isotropic Gaussian random field $\mathbf{W}$
- ▶ γ : spatio-temporal variogram of $\mathbf{W}$

DEPENDENCE MEASURES

Let $\Lambda_S \subset \mathbb{R}_+^2$ and $\Lambda_T \subset \mathbb{R}_+$ be sets of spatial and temporal lags respectively.

Spatio-temporal extremogram

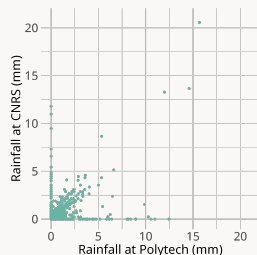
For all $\mathbf{h} \in \Lambda_S, \tau \in \Lambda_T$,

$$\chi(\mathbf{h}, \tau) = \lim_{q \rightarrow 1} \mathbb{P}(X_{\mathbf{s},t}^* > q \mid X_{\mathbf{s}+\mathbf{h},t+\tau}^* > q),$$

with $q \in [0, 1[$ and $X_{\mathbf{s},t}^*$ the uniform margins.

Spatio-temporal variogram γ

$$\gamma(\mathbf{h}, \tau) = \frac{1}{2} \text{Var} (W_{\mathbf{s},t} - W_{\mathbf{s}+\mathbf{h},t+\tau}), \mathbf{h} \in \Lambda_S, \tau \in \Lambda_T$$



DEPENDENCE MEASURES

Let $\Lambda_S \subset \mathbb{R}_+^2$ and $\Lambda_T \subset \mathbb{R}_+$ be sets of spatial and temporal lags respectively.

Spatio-temporal extremogram

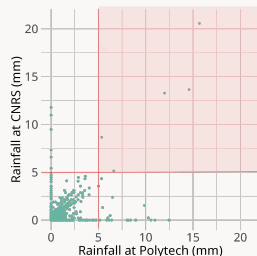
For all $\mathbf{h} \in \Lambda_S, \tau \in \Lambda_T$,

$$\chi(\mathbf{h}, \tau) = \lim_{q \rightarrow 1} \mathbb{P}(X_{\mathbf{s},t}^* > q \mid X_{\mathbf{s}+\mathbf{h},t+\tau}^* > q),$$

with $q \in [0, 1[$ and $X_{\mathbf{s},t}^*$ the uniform margins.

Spatio-temporal variogram γ

$$\gamma(\mathbf{h}, \tau) = \frac{1}{2} \text{Var} (W_{\mathbf{s},t} - W_{\mathbf{s}+\mathbf{h},t+\tau}), \mathbf{h} \in \Lambda_S, \tau \in \Lambda_T$$



Spatio-temporal extremogram of a Brown-Resnick process

Let $\mathbf{h} \in \Lambda_{\mathcal{S}}$ and $\tau \in \Lambda_{\mathcal{T}}$. We have

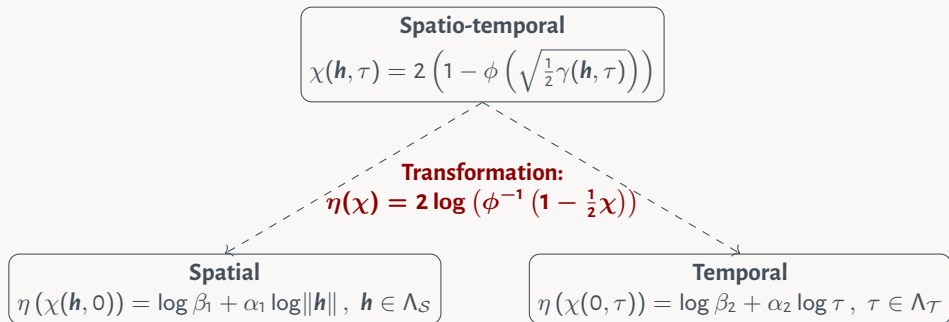
$$\chi(\mathbf{h}, \tau) = 2 \left(1 - \phi \left(\sqrt{\frac{1}{2} \gamma(\mathbf{h}, \tau)} \right) \right)$$

with ϕ the std normal c.d.f. and γ the variogram of $\mathbf{W}$.

Dependence model framework: BUHL et al. 2019

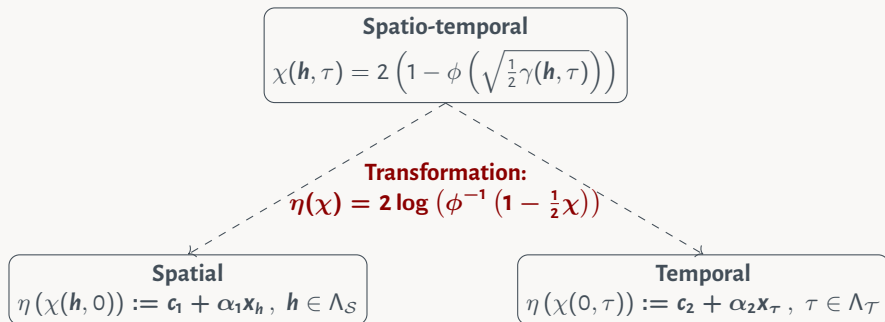
SPATIO-TEMPORAL DEPENDENCE MODELING

Case of additive separability: $\frac{\gamma(h, \tau)}{2} = \beta_1 \|h\|^{\alpha_1} + \beta_2 \tau^{\alpha_2}$, $0 < \alpha_1, \alpha_2 \leq 2$, $\beta_1, \beta_2 > 0$



SPATIO-TEMPORAL DEPENDENCE MODELING

Case of additive separability: $\frac{\gamma(h, \tau)}{2} = \beta_1 \|h\|^{\alpha_1} + \beta_2 \tau^{\alpha_2}, 0 < \alpha_1, \alpha_2 \leq 2, \beta_1, \beta_2 > 0$



Weighted Least Squares Estimation (WLSE)

$$\begin{pmatrix} \hat{c}_i \\ \hat{\alpha}_i \end{pmatrix} = \operatorname{argmin}_{c_i, \alpha_i} \sum_x w_x (\eta(\hat{\chi}) - (c_i + \alpha_i x))^2$$

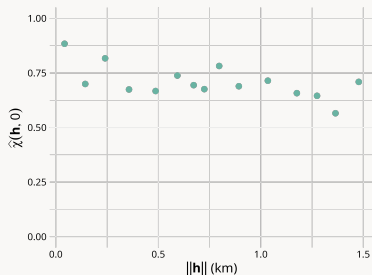
SPATIAL DEPENDENCE ESTIMATION

Empirical spatial extremogram

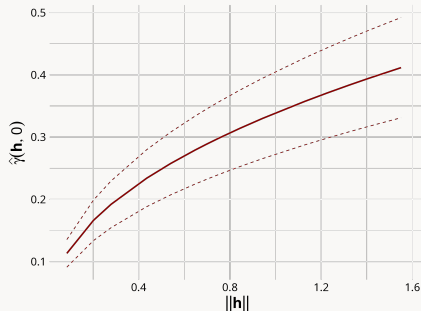
For a fixed $t \in \mathcal{T}$ and q a high quantile,

$$\hat{\chi}_q^{(t)}(\mathbf{h}, 0) = \frac{\frac{1}{|N_{\mathbf{h}}|} \sum_{i,j \mid (\mathbf{s}_i, \mathbf{s}_j) \in N_{\mathbf{h}}} \mathbb{1}_{\{X_{\mathbf{s}_i, t}^* > q, X_{\mathbf{s}_j, t}^* > q\}}}{\frac{1}{|\mathcal{S}|} \sum_{i=1}^{|\mathcal{S}|} \mathbb{1}_{\{X_{\mathbf{s}_i, t}^* > q\}}},$$

where $C_{\mathbf{h}}$ are equifrequent distance classes and $N_{\mathbf{h}} = \{(\mathbf{s}_i, \mathbf{s}_j) \in \mathcal{S}^2 \mid \|\mathbf{s}_i - \mathbf{s}_j\| \in C_{\mathbf{h}}\}$.



Transformation and WLSE



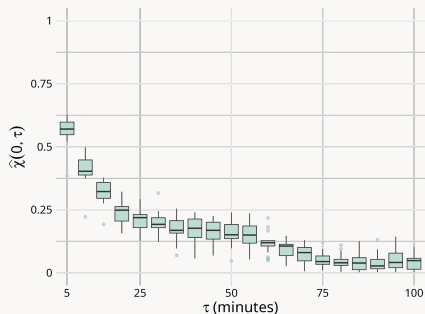
Spatial variogram $\hat{\gamma}(\mathbf{h}, 0) = 2\hat{\beta}_1 \|\mathbf{h}\|^{\hat{\alpha}_1}$

TEMPORAL DEPENDENCE ESTIMATION

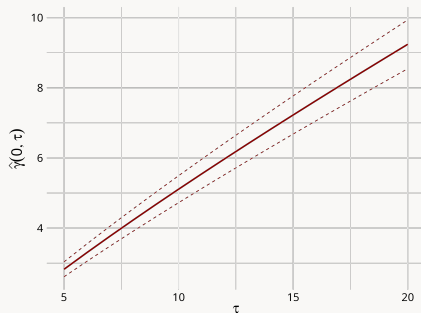
Empirical temporal extremogram

For a location $s \in \mathcal{S}$, a high quantile q and $t_k \in \{t_1, \dots, t_T\}$,

$$\hat{\chi}_q^{(s)}(\mathbf{0}, \tau) = \frac{\frac{1}{T-\tau} \sum_{k=1}^{T-\tau} \mathbb{1}_{\{X_{s,t_k}^* > q, X_{s,t_k+\tau}^* > q\}}}{\frac{1}{T} \sum_{k=1}^T \mathbb{1}_{\{X_{s,t_k}^* > q\}}}$$



Transformation and WLSE



Temporal variogram $\hat{\gamma}(\mathbf{0}, \tau) = 2\hat{\beta}_2\tau^{\hat{\alpha}_2}$

CONSIDERING ADVECTION

Advection vector $\mathbf{V}$

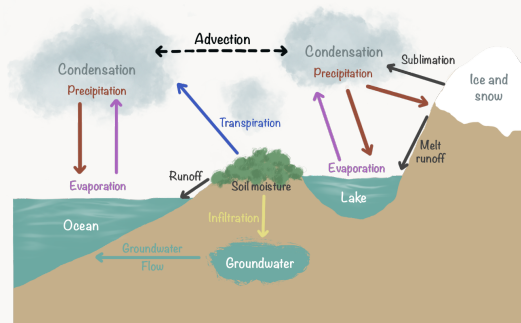
- Horizontal transport of air masses
- To relax the separability assumption

Lagrangian/Eulerian variogram

$$\gamma_L(\mathbf{h}, \tau) = \gamma(\mathbf{h} - \tau \mathbf{V}, \tau)$$

Dependence model

$$\frac{1}{2} \gamma_L(\mathbf{h}, \tau) = \beta_1 \|\mathbf{h} - \tau \mathbf{V}\|^{\alpha_1} + \beta_2 \tau^{\alpha_2}$$



Hydrologic cycle

CONSIDERING ADVECTION - ESTIMATION

Parameter optimization of $\Theta = (\beta_1, \beta_2, \alpha_1, \alpha_2, \mathbf{V})$

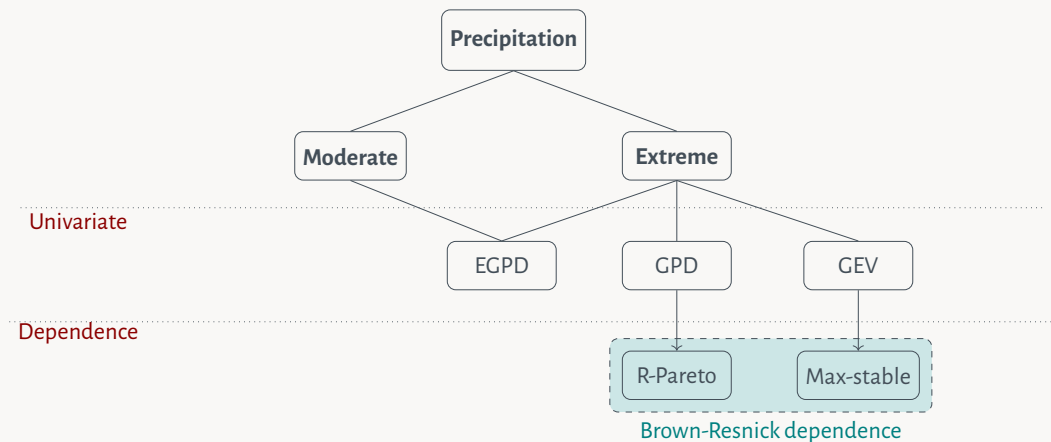
Excesses: for all spatial pairs $(\mathbf{s}_i, \mathbf{s}_j)$,

$$k_{ij} = \sum_{t=1}^n \mathbb{1}_{\{X_{\mathbf{s}_i,t} > q, X_{\mathbf{s}_j,t} > q\}} \mid n_j \sim \mathcal{B}(n_j, \chi_{ij}, \Theta)$$

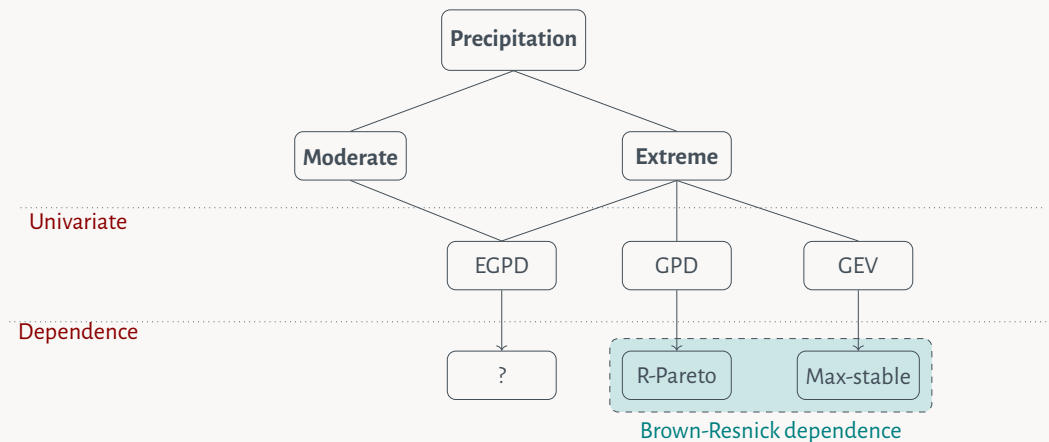
Composite log-likelihood:

$$l_C(\Theta) \approx \sum_{ij} k_{ij} \log \chi_{ij}, \Theta + (n_j - k_{ij}) \log(1 - \chi_{ij}, \Theta)$$

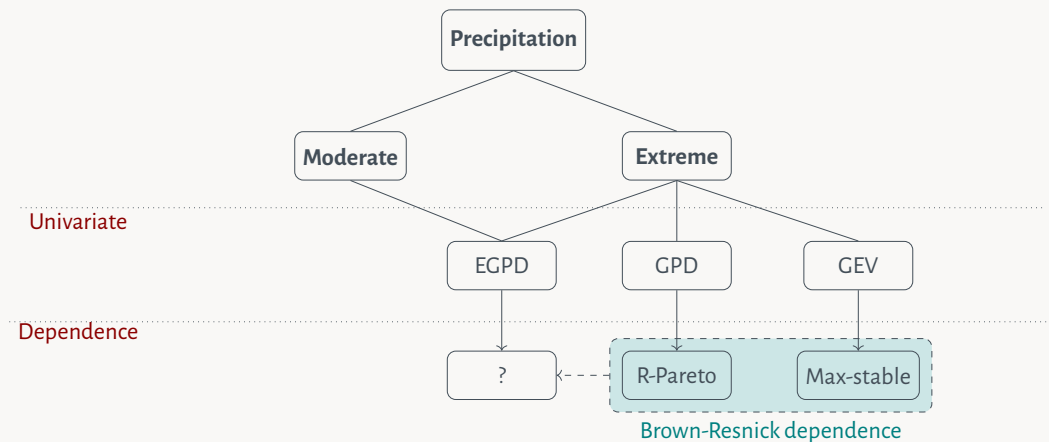
CONCLUSION



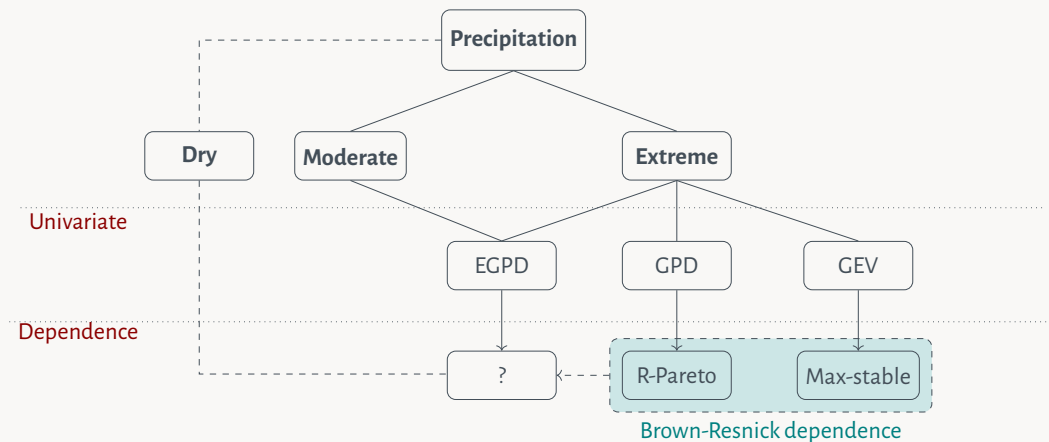
CONCLUSION



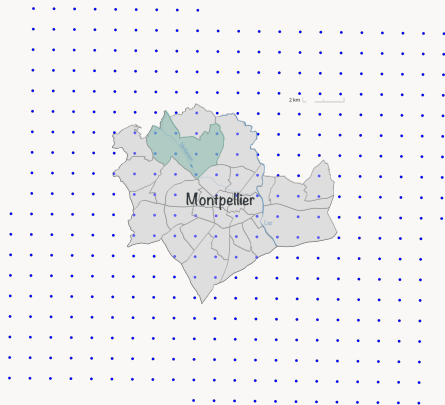
CONCLUSION



CONCLUSION



ADDITIONAL DATA



- **Source:** COMEPHORE, Météo France
- **Time period:** [1997, 2023]
- **Temporal resolution:** Every hour
- **Spatial resolution:** 1 km²

$$\mathcal{S} = \{400 \text{ pixels}\} \subset \mathbb{R}^2 \text{ and } \mathcal{T} \subset \mathbb{R}_+$$

REFERENCES

- BROWN, Bruce M. and Sidney I. RESNICK (1977). "Extreme values of independent stochastic processes". In: *Journal of Applied Probability*. DOI: 10.2307/3213346.
- BUHL, Sven et al. (2019). "Semiparametric estimation for isotropic max-stable space-time processes". In: *Bernoulli*.
- FINAUD-GUYOT, Pascal et al. (2023). *Rainfall data collected by the HSM urban observatory (OMSEV)*. DOI: 10.23708/67LC36.
- HARUNA, Abubakar, Juliette BLANCHET, and Anne-Catherine FAVRE (2023). *Modeling Areal Precipitation Hazard: A Data-driven Approach to Model Intensity-Duration-Area-Frequency Relationships using the Full Range of Non-Zero Precipitation in Switzerland*. preprint. Preprints. DOI: 10.22541/essoar.169111775.53035997/v1.
- NAVEAU, Philippe et al. (2016). "Modeling jointly low, moderate, and heavy rainfall intensities without a threshold selection". In: *Water Resources Research*. DOI: 10.1002/2015WR018552.

MODEL VALIDATION - SIMULATIONS

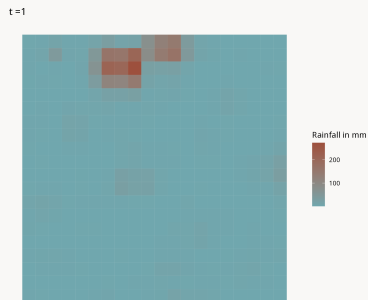
Brown-Resnick simulations

- Spatial: $\mathcal{S} = \{400 \text{ sites}\}$, $\mathcal{T} = \{1, \dots, 50\}$, $|\Lambda_{\mathcal{S}}| = 10$ and $|\Lambda_{\mathcal{T}}| = 10$
- Temporal: $\mathcal{S} = \{25 \text{ sites}\}$, $\mathcal{T} = \{1, \dots, 300\}$, $|\Lambda_{\mathcal{S}}| = 10$ and $|\Lambda_{\mathcal{T}}| = 10$

	True	Mean	RMSE	MAE
$\hat{\beta}_1$	0.4	0.524	0.138	0.126
$\hat{\alpha}_1$	1.5	1.507	0.120	0.088
$\hat{\beta}_2$	0.2	0.259	0.093	0.074
$\hat{\alpha}_2$	1	0.873	0.149	0.128

Parameter estimates for 100 realisations

True variogram: $\frac{1}{2}\gamma(\mathbf{h}, \tau) = 0.4\|\mathbf{h}\|^{3/2} + 0.2\tau$



CONSIDERING ADVECTION

Advection vector $\mathbf{V}$

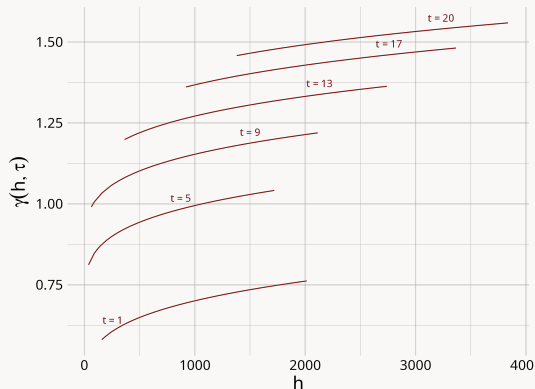
- ▶ Horizontal transport of air masses
- ▶ To relax the separability assumption

Lagrangian/Eulerian variogram

$$\gamma_L(\mathbf{h}, \tau) = \gamma(\mathbf{h} - \tau \mathbf{V}, \tau)$$

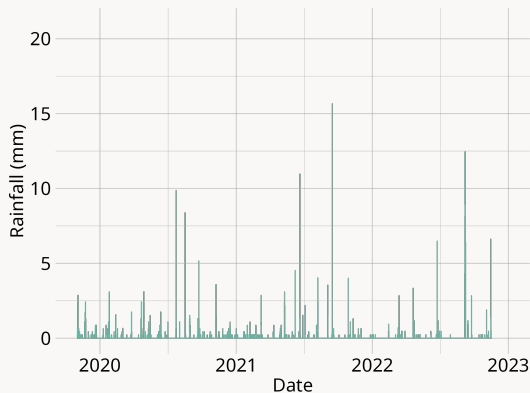
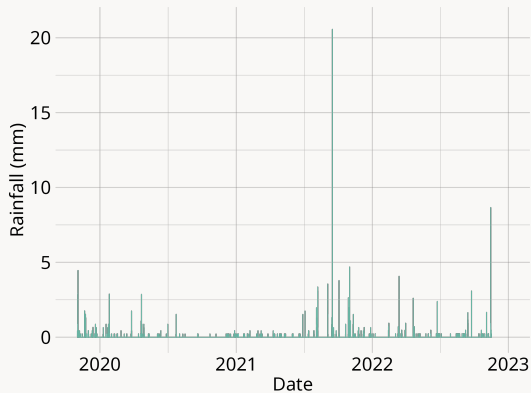
Dependence model

$$\frac{1}{2} \gamma_L(\mathbf{h}, \tau) = \beta_1 \|\mathbf{h} - \tau \mathbf{V}\|^{\alpha_1} + \beta_2 \tau^{\alpha_2}$$



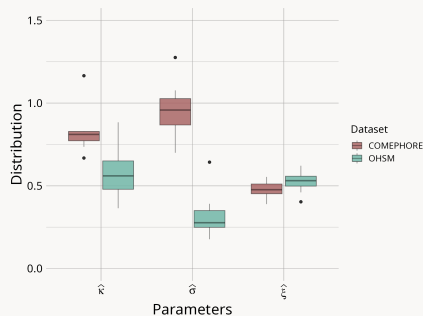
Spatial variogram with a constant advection
 $\mathbf{V} = (0.001, 45)^T$ on OHSM data

RAINFALL DATA - OHSM

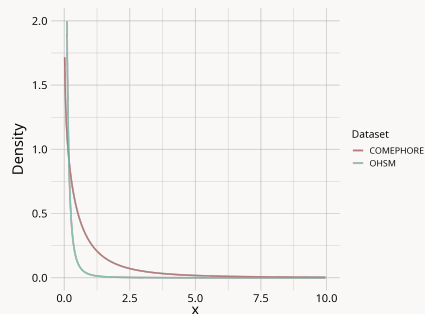


Rainfall amounts on CNRS and Polytech rain gauges

Left-censoring: selected according to the NRMSE criterion for each site individually³



Parameter estimates



Density with mean parameters

³HARUNA, BLANCHET, and FAVRE 2023