Spatio-temporal modeling of urban extreme rainfall events at high resolution

SWGEN, Grenoble, December 3, 2025

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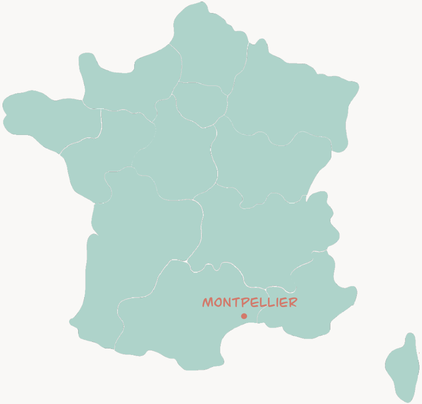
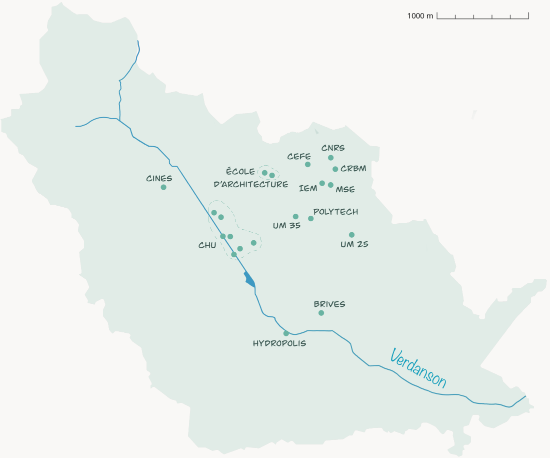
Floods in Montpellier, September 2022 and August 2015 (*Midi Libre*)

- Mediterranean events, localized rainfall
- Urban area, flood risks

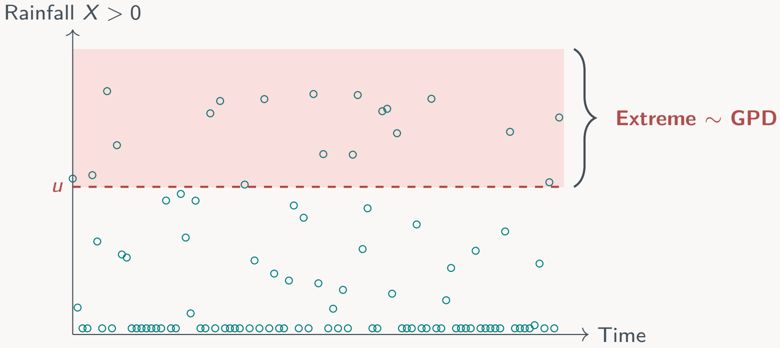




- **Study area:** Verdanson water catchment
- **Source:** Urban observatory of HydroScience Montpellier (HSM)[2]
- **Time period:** [Sept.2019, Jan.2025[
- **Temporal resolution:** 5 minutes
- **Spatial resolution:** 77 m to 2259 m

$\mathcal{S} = \{\textbf{20 rain gauges}\} \subset \mathbb{R}^2$ and $\mathcal{T} \subset \mathbb{R}_+$

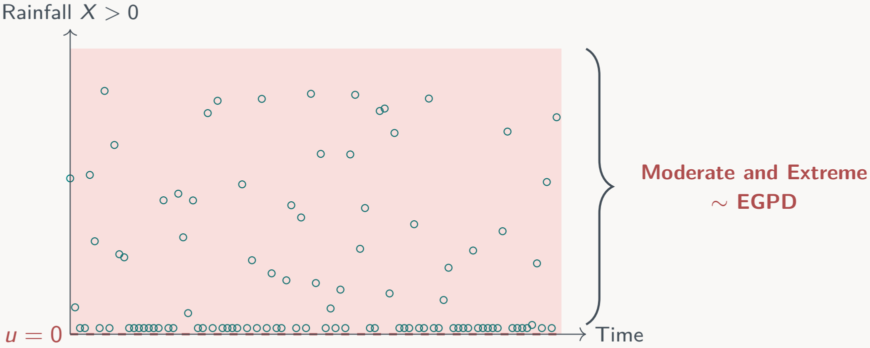
[1] Observ. Montpellierain et au Sud de l'Eau dans la Ville
[2] Finaud-Guyot et al., 2023



- **Generalized Pareto Distribution (GPD)**[1]

$$X \mid X > u \sim \underbrace{H_\xi}_{\text{GPD}(\xi,\sigma,u)} \text{ with } \xi \in \mathbb{R},\ \sigma > 0.$$

[1] Pickands III, 1975

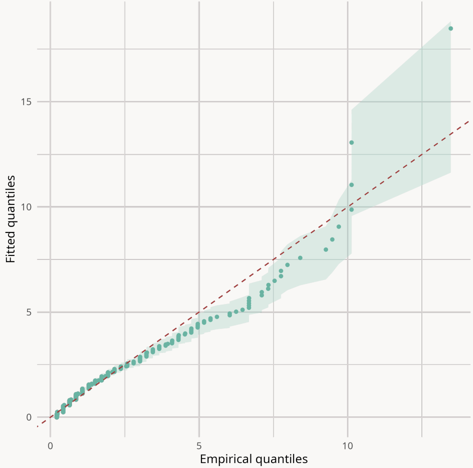


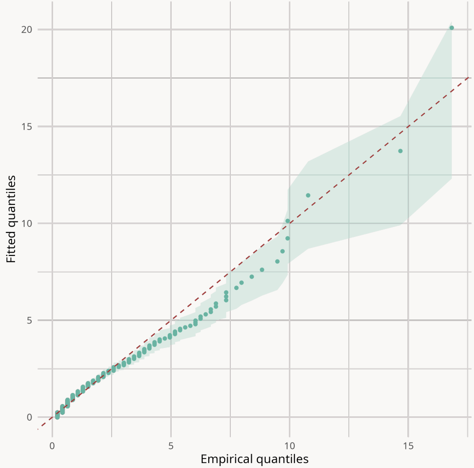
- **Extended G**eneralized **P**areto **D**istribution (**EGPD**)[2]

$$X \sim \underbrace{G(H_\xi)}_{\text{EGPD}(\xi,\sigma,\kappa)} \text{ with } G(x) = x^\kappa,\ \kappa > 0$$

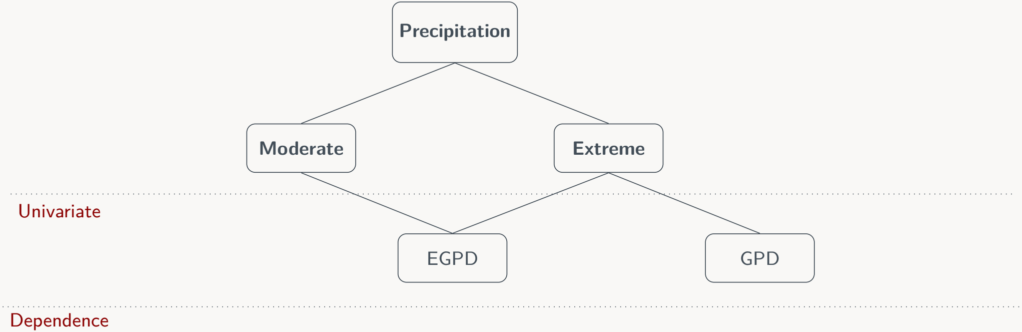
[2] Naveau et al., 2016

EGPD fitting





EGPD fitting for two rain gauges, CEFE (left) and IEM (right) with left-censoring and 95% CI


Precipitation
Moderate
Extreme
Univariate
EGPD
GPD
Dependence

Rainfall random field: $\boldsymbol{X} = \{X_{\boldsymbol{s},t},\ (\boldsymbol{s}, t) \in \mathcal{S} \times \mathcal{T}\}$

Let $\Lambda_{\mathcal{S}} \subset \mathbb{R}_+^2$ and $\Lambda_{\mathcal{T}} \subset \mathbb{R}_+$ be sets of spatial and temporal lags respectively.

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Variogram (Matheron, 1963)

$$\gamma(\boldsymbol{h}, \tau) = \frac{1}{2}\mathrm{Var}(X_{\boldsymbol{s},t} - X_{\boldsymbol{s}+\boldsymbol{h},t+\tau})$$

- Quantifies variability
- Higher $\gamma(\boldsymbol{h}, \tau) \to$ weaker dependence

$\boldsymbol{h} \in \Lambda_{\mathcal{S}},\ \tau \in \Lambda_{\mathcal{T}}$

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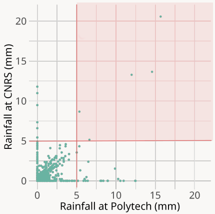
Extremogram (Davis and Mikosch, 2009)

$$\chi(\boldsymbol{h},\tau) = \lim_{q\to 1} \mathbb{P}(X^*_{\boldsymbol{s},t} > q \mid X^*_{\boldsymbol{s}+\boldsymbol{h},t+\tau} > q)$$

- Measures tail dependence
- Higher $\chi(\boldsymbol{h},\tau) \to$ stronger dependence

$\boldsymbol{h} \in \Lambda_{\mathcal{S}}$, $\tau \in \Lambda_{\mathcal{T}}$, $X^*_{\boldsymbol{s},t}$ uniform margins.

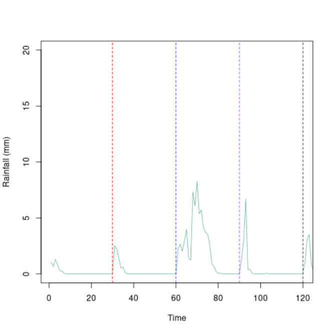


Definition (de Fondeville and Davison, 2018)

For all $\boldsymbol{s} \in \mathcal{S}$ and $t \in \mathcal{T}$, a risk function $r(\boldsymbol{X}) = X_{\boldsymbol{s}_0, t_0}$,

$$u^{-1} X_{\boldsymbol{s},t} \mid X_{\boldsymbol{s}_0, t_0} > u \xrightarrow{d} Y_{\boldsymbol{s},t} \quad \text{with} \quad Y_{\boldsymbol{s},t} = R e^{W_{\boldsymbol{s},t} - W_{\boldsymbol{s}_0, t_0} - \gamma(\boldsymbol{s} - \boldsymbol{s}_0, t - t_0)},$$

where $(\boldsymbol{s}_0, t_0)$ is a space-time location, u is a high threshold, $R \sim \text{Pareto}(1)$, $W_{\boldsymbol{s},t}$ is a Gaussian process.



Random simulation

Spatio-temporal extremogram with a Brown-Resnick dependence

Let $\boldsymbol{h} \in \Lambda_{\mathcal{S}}$ and $\tau \in \Lambda_{\mathcal{T}}$. We have

$$\chi(\boldsymbol{h}, \tau) = 2\left(1 - \phi\left(\sqrt{\frac{1}{2}\gamma(\boldsymbol{h}, \tau)}\right)\right)$$

with ϕ the std normal c.d.f. and γ the variogram of $\boldsymbol{W}$.

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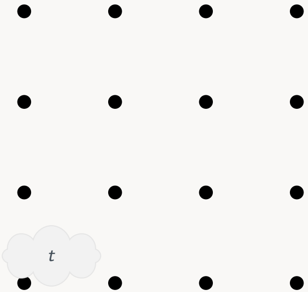
Separable model: Fractional Brownian motion with additive separability.

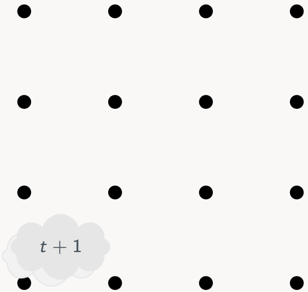
$$\frac{\gamma(\boldsymbol{h}, \tau)}{2} = \beta_1 \|\boldsymbol{h}\|^{\alpha_1} + \beta_2 |\tau|^{\alpha_2}$$

with $0 < \alpha_1, \alpha_2 \leq 2$, $\beta_1, \beta_2 > 0$.

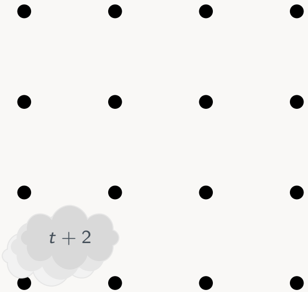
Separable model



Separable model



Separable model



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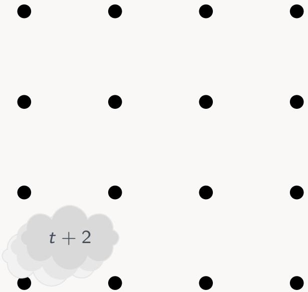
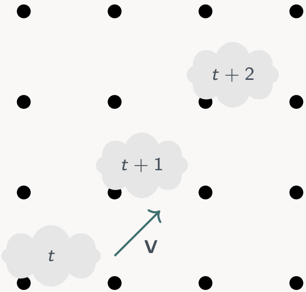


Non-separable model



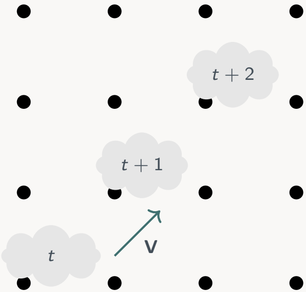
Towards more realistic modeling: introduce advection $\boldsymbol{V}$ to relax separability

$$\gamma_L(\boldsymbol{h},\tau) = \gamma(\boldsymbol{h} - \tau\boldsymbol{V}, \tau)$$

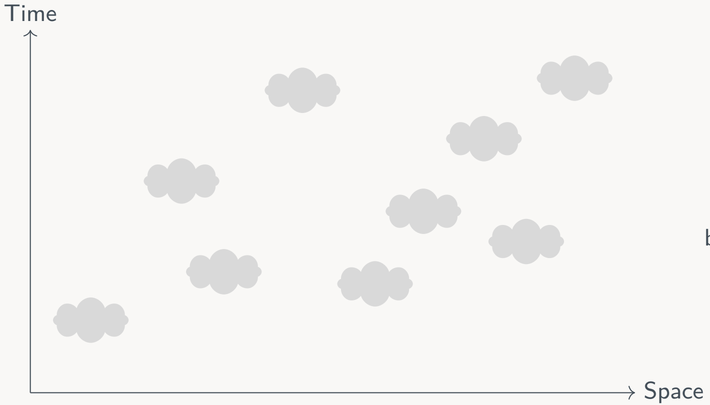
$$\Rightarrow \quad \frac{1}{2}\gamma_L(\boldsymbol{h},\tau) = \beta_1\|\boldsymbol{h} - \tau\boldsymbol{V}\|^{\alpha_1} + \beta_2|\tau|^{\alpha_2}$$

- Parameters: $\boldsymbol{\Theta} = (\beta_1, \beta_2, \alpha_1, \alpha_2, \boldsymbol{V})$

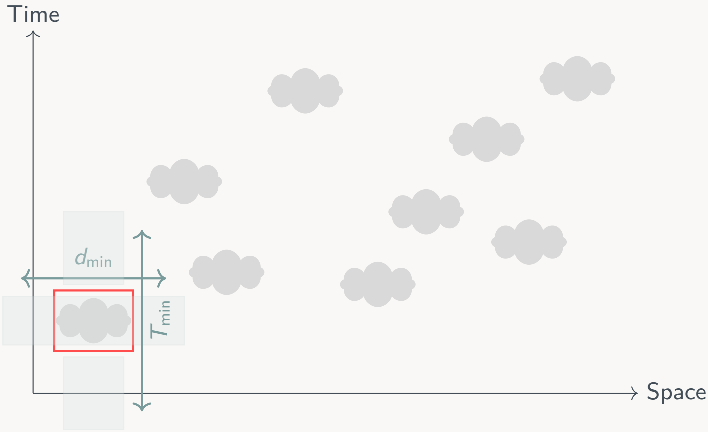
Non-separable model



All rainfall events



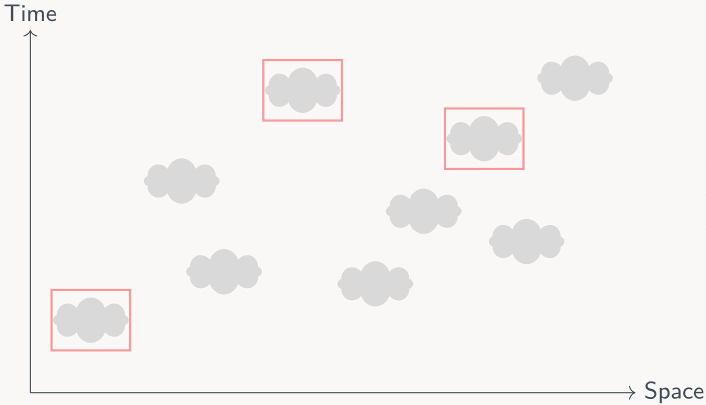
r-Pareto process:
Each episode is characterized by $(\mathbf{s}_0, t_0)$ for which $X_{\mathbf{s}_0, t_0} > u$



Episode selection:

Only episodes separated by
- spatial distance $\geq d_{\text{min}}$
- temporal gap $\geq T_{\text{min}}$

$\Rightarrow$ reduces dependence between selected episodes $\in \mathcal{E}$.



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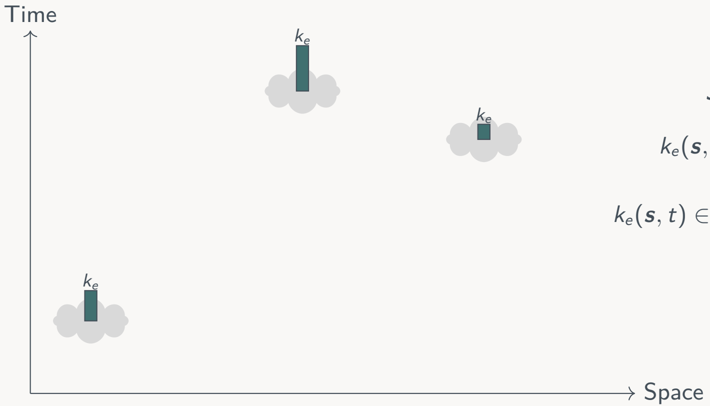
Count joint exceedances per episode

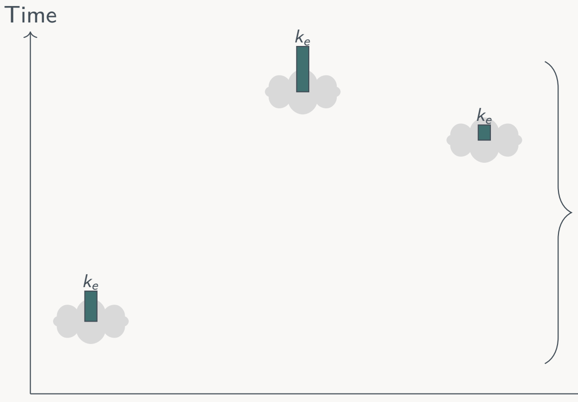


Joint exceedances

$$k_e(\boldsymbol{s}, t) = \mathbb{1}_{\{X_{\boldsymbol{s}_0, t_0} > u,\ X_{\boldsymbol{s}, t} > u\}},$$
with $(\boldsymbol{s}, t) \in \mathcal{N}(\boldsymbol{h}, \tau)$

$$k_e(\boldsymbol{s}, t) \in [0, 1] \sim \text{Bernoulli}(\chi_\Theta(h, \tau))$$

with $\mathcal{N}(\boldsymbol{h}, \tau) = \{(\boldsymbol{s}, t) \in \mathcal{S} \times \mathcal{T} \mid \boldsymbol{s} - \boldsymbol{s}_0 = \boldsymbol{h},\ t - t_0 = \tau\}$



Total joint exceedances:

$$k_e(\boldsymbol{s}, t) \in [0, 1] \sim \text{Bernoulli}(\chi_\Theta(h, \tau))$$

$$\Rightarrow K_{\boldsymbol{h},\tau} = \sum_{e \in \mathcal{E}} \sum_{(\boldsymbol{s},t) \in \mathcal{N}(\boldsymbol{h},\tau)} k_e(\boldsymbol{s}, t)$$

Composite Binomial likelihood:

$$\mathcal{B}\left(N_{\text{tot}}(\boldsymbol{h}),\ \chi_{\boldsymbol{\Theta}}(\boldsymbol{h}, \tau)\right)$$

with $\mathcal{N}(\boldsymbol{h}, \tau) = \{(\boldsymbol{s}, t) \in \mathcal{S} \times \mathcal{T} \mid \boldsymbol{s} - \boldsymbol{s}_0 = \boldsymbol{h},\ t - t_0 = \tau\}$
and $N_{\text{tot}}(\boldsymbol{h}) = |\mathcal{E}| \times \#\{\boldsymbol{s} \in \mathcal{S} \mid \boldsymbol{s} - \boldsymbol{s}_0 = \boldsymbol{h}\}$

Bernoulli contributions (large u):

$$k_e(\boldsymbol{s}, t) = \mathbf{1}_{\{X_{\boldsymbol{s}_0,t_0}>u,\ X_{\boldsymbol{s},t}>u\}}, \qquad (\boldsymbol{s}, t) \in \mathcal{N}(\boldsymbol{h}, \tau),$$

each treated as

$$k_e(\boldsymbol{s}, t) \sim \mathrm{Bernoulli}\Big(\chi_\Theta(\boldsymbol{h}, \tau)\Big).$$

Composite Binomial likelihood:

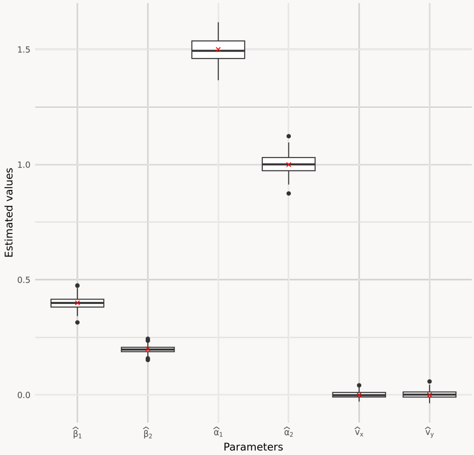
$$\mathcal{B}(N_{\text{tot}}(\boldsymbol{h}),\ \chi_\Theta(\boldsymbol{h}, \tau)), \qquad N_{\text{tot}}(\boldsymbol{h}) = |\mathcal{E}|\,\#\{\boldsymbol{s} \in \mathcal{S} : \boldsymbol{s} - \boldsymbol{s}_0 = \boldsymbol{h}\}.$$

Composite log-likelihood

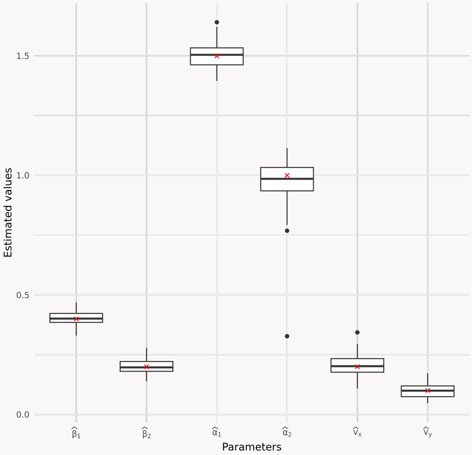
$$\ell_C(\Theta) \propto \sum_{e\in\mathcal{E}} \sum_{(\boldsymbol{h},\tau)\in\Lambda_{\mathcal{S}}\times\Lambda_{\mathcal{T}}} \sum_{(\boldsymbol{s},t)\in\mathcal{N}(\boldsymbol{h},\tau)} k_e(\boldsymbol{s}, t) \log \chi_\Theta(\boldsymbol{h}, \tau) + (1 - k_e(\boldsymbol{s}, t)) \log(1 - \chi_\Theta(\boldsymbol{h}, \tau)).$$

- Optimization via maximization of $\ell_C(\Theta)$.

Validation: constant advection



Without advection



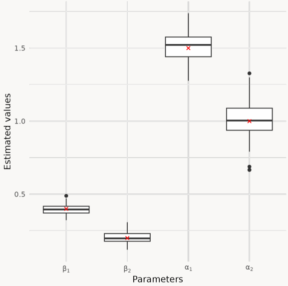
With advection

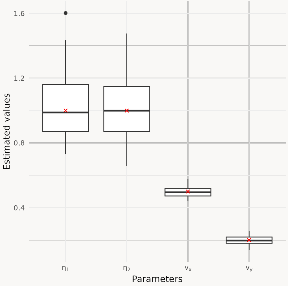
50 simulations, 500 replicates, 25 sites, 30 time steps

Episode-wise advection to global model

$$\mathbf{V}^{\mathrm{emp}} = \begin{bmatrix} \mathbf{V}^{(1)} \\ \mathbf{V}^{(2)} \\ \vdots \\ \mathbf{V}^{(N)} \end{bmatrix} \in \mathbb{R}^{N\times 2} \xrightarrow{\text{global model}} \underbrace{\mathbf{V}^{\mathrm{final}} = \eta_1 \,\mathrm{sign}(\mathbf{V}^{\mathrm{emp}}) \odot \left|\mathbf{V}^{\mathrm{emp}}\right|^{\odot \eta_2} \in \mathbb{R}^{N\times 2}}_{\text{unified transformation} \,\Rightarrow\, \eta \text{ to estimate}}$$

one component $=$ one episode





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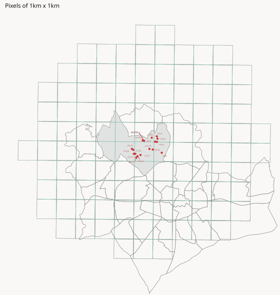
one component = one episode

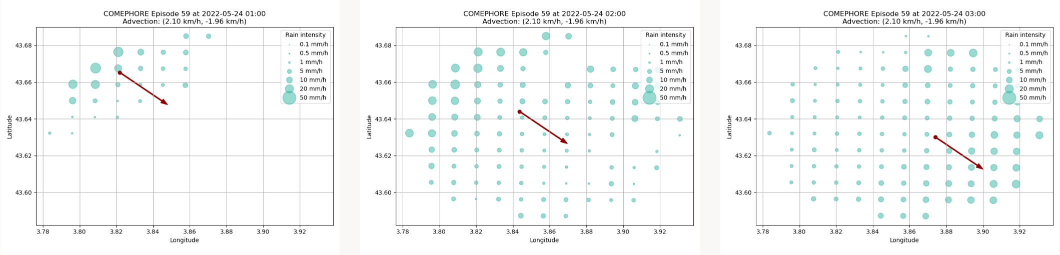
Challenge:
OMSEV data $\Rightarrow$ limited information for advection

Approach:
Fusion COMEPHORE–OMSEV $\rightarrow$ more reliable advection
Advection classes $\rightarrow$ class-dependent $\eta = (\eta_1, \eta_2)$



COMEPHORE pixels
Météo France

For one episode (COMEPHORE):

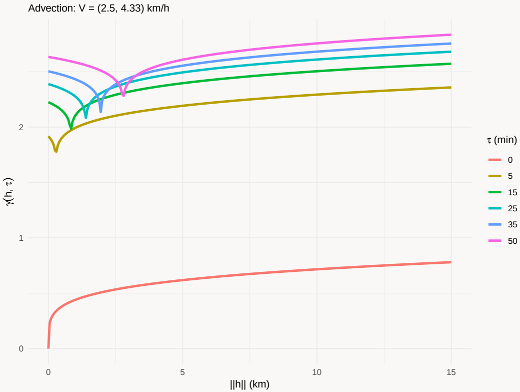


$t_0 - 1$ $\qquad$ t_0 $\qquad$ $t_0 + 1$

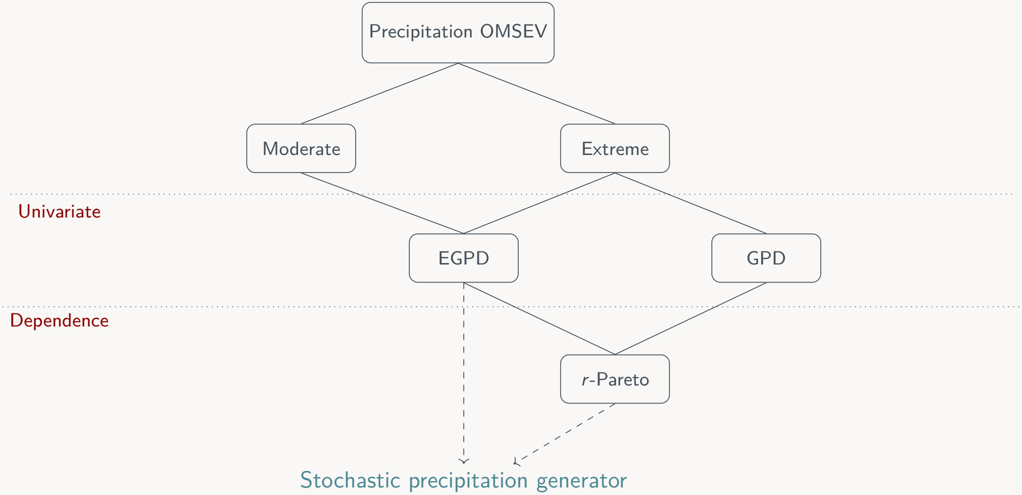
$\boldsymbol{V}^{\mathrm{emp}}$ is estimated from **rain storm barycenter displacement** within a time window.

OMSEV results

- Advection group: relatively strong ($\|\boldsymbol{V}\| > 2$ km/h), north direction $\Rightarrow$ 46 episodes
- Fixed η from COMEPHORE data: $\widehat{\eta}_1 = 0.667$, $\widehat{\eta}_2 = 1.757$ (km/h).
- OMSEV estimates (m/5 min): $\widehat{\beta}_1 = 0.066$, $\widehat{\beta}_2 = 0.747$, $\widehat{\alpha}_1 = 0.480$, $\widehat{\alpha}_2 = 0.691$



Estimated variogram with $\boldsymbol{V} = (2.5, 4.3)$ km/h


Precipitation OMSEV
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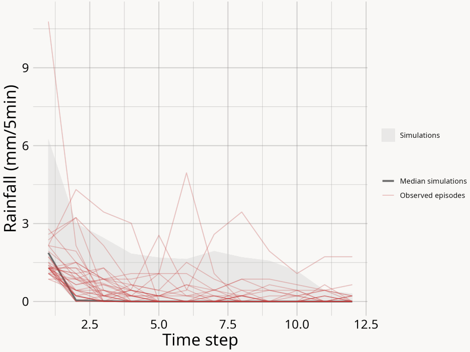
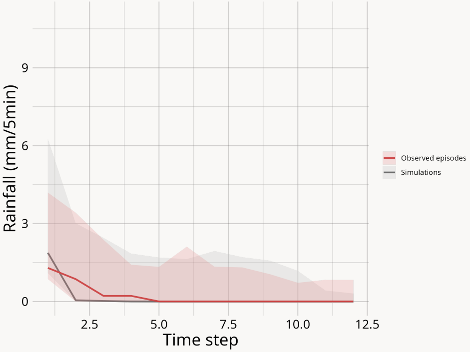
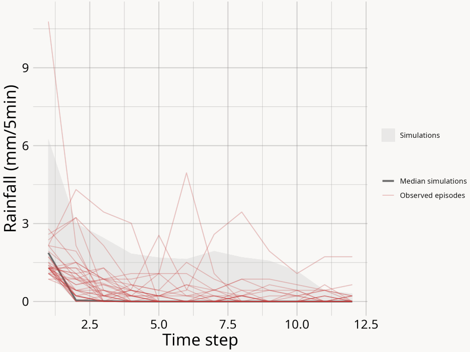


Moderate advection intensity
1000 simulations vs 34 observations



Moderate advection intensity
95% CI



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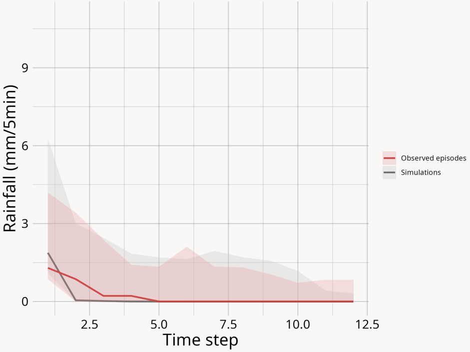
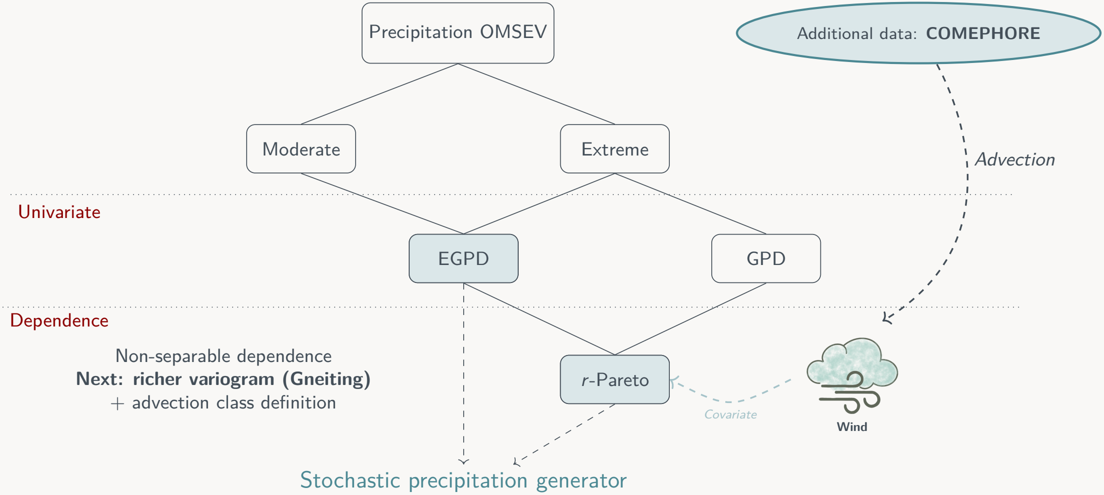


Moderate advection intensity
95% CI

Challenge:

- Limited information to estimate advection: incorporate wind data
- Advection class definition still needs refinement

Conclusion & Outlook



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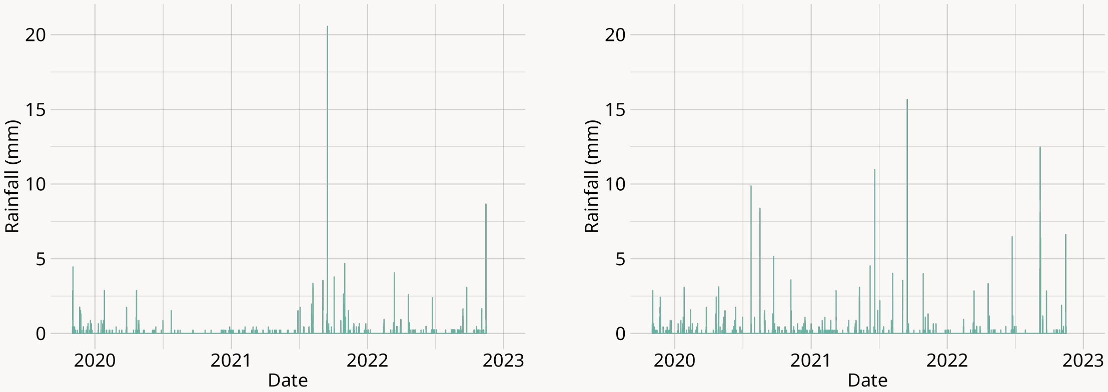
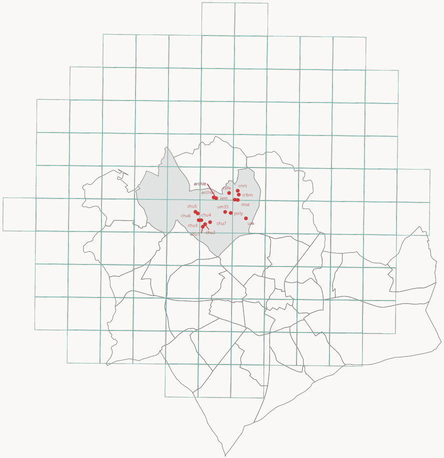


Rainfall amounts on CNRS and Polytech rain gauges

Pixels of 1km x 1km



- **Source:** COMEPHORE, Météo France
- **Time period:** $[1997, 2023[$
- **Temporal resolution:** Every hour
- **Spatial resolution:** 1 km^2

More consistent data: Both datasets + Neural Network Downscaling.

Generalized Pareto Distribution → **Extended GPD**[1]

$$\overline{H}_\xi\left(\frac{x-u}{\sigma}\right) = \begin{cases} \left(1+\xi\frac{x-u}{\sigma}\right)_+^{-1/\xi} & \text{if } \xi \neq 0\,, \\ e^{-\frac{x-u}{\sigma}} & \text{if } \xi = 0\,, \end{cases}$$

where $a_+ = \max(a, 0)$, $\sigma > 0$, $x - u > 0$

- Models extreme precipitation
- Depends on a threshold choice

$$F(x) = G\left(H_\xi\left(\frac{x}{\sigma}\right)\right),$$

where $G(x) = x^\kappa$, $\kappa > 0$

- Models **moderate and extreme** precipitation
- Avoids a threshold choice

[1] Naveau et al., 2016

Estimation of the separable variogram parameters

Case of additive separability: $\frac{\gamma(\boldsymbol{h},\tau)}{2} = \beta_1 \|\boldsymbol{h}\|^{\alpha_1} + \beta_2 |\tau|^{\alpha_2}, \quad 0 < \alpha_1, \alpha_2 \leq 2\,, \; \beta_1, \beta_2 > 0$

Spatio-temporal

$$\chi(\boldsymbol{h}, \tau) = 2\left(1 - \phi\left(\sqrt{\tfrac{1}{2}\gamma(\boldsymbol{h}, \tau)}\right)\right)$$

Transformation:

$$\zeta(\chi) = 2\log\left(\phi^{-1}\left(1 - \tfrac{1}{2}\chi\right)\right)$$

Spatial

$$\zeta\left(\chi(\boldsymbol{h}, 0)\right) = \log\beta_1 + \alpha_1 \log\|\boldsymbol{h}\|\,, \;\; \boldsymbol{h} \in \Lambda_{\mathcal{S}}$$

Temporal

$$\zeta\left(\chi(0, \tau)\right) = \log\beta_2 + \alpha_2 \log\tau\,, \;\; \tau \in \Lambda_{\mathcal{T}}$$

Estimation of the separable variogram parameters

Case of additive separability: $\frac{\gamma(\boldsymbol{h},\tau)}{2} = \beta_1\|\boldsymbol{h}\|^{\alpha_1} + \beta_2|\tau|^{\alpha_2}, \quad 0 < \alpha_1, \alpha_2 \leq 2\,, \ \beta_1, \beta_2 > 0$

Spatio-temporal

$$\chi(\boldsymbol{h},\tau) = 2\left(1 - \phi\left(\sqrt{\tfrac{1}{2}\gamma(\boldsymbol{h},\tau)}\right)\right)$$

Transformation:

$$\zeta(\chi) = 2\log\left(\phi^{-1}\left(1 - \tfrac{1}{2}\chi\right)\right)$$

Spatial

$$\eta\left(\chi(\boldsymbol{h},0)\right) := c_1 + \alpha_1 x_h\,, \quad \boldsymbol{h} \in \Lambda_{\mathcal{S}}$$

Temporal

$$\eta\left(\chi(0,\tau)\right) := c_2 + \alpha_2 x_\tau\,, \quad \tau \in \Lambda_{\mathcal{T}}$$

Weighted Least Squares Estimation (WLSE)

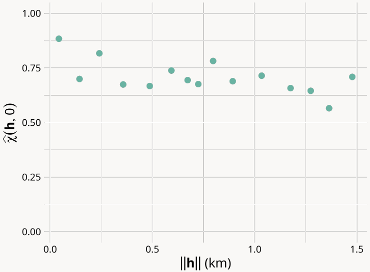
$$\begin{pmatrix} \widehat{c_i} \\ \widehat{\alpha_i} \end{pmatrix} = \operatorname{argmin}_{c_i,\alpha_i} \sum_x w_x \left(\zeta\left(\widehat{\chi}\right) - (c_i + \alpha_i x)\right)^2$$

Empirical spatial extremogram

For a fixed $t \in \mathcal{T}$ and q a high quantile,

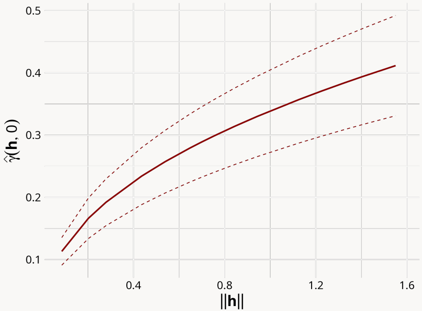
$$\widehat{\chi}_q^{(t)}(\boldsymbol{h}, 0) = \frac{\frac{1}{|N_{\boldsymbol{h}}|} \sum_{i,j \,|\, (\boldsymbol{s}_i, \boldsymbol{s}_j) \in N_{\boldsymbol{h}}} \mathbb{1}_{\{X^*_{\boldsymbol{s}_i,t} > q \,, X^*_{\boldsymbol{s}_j,t} > q\}}}{\frac{1}{|\mathcal{S}|} \sum_{i=1}^{|\mathcal{S}|} \mathbb{1}_{\{X^*_{\boldsymbol{s}_i,t} > q\}}},$$

where $C_{\boldsymbol{h}}$ are equifrequent distance classes and $N_{\boldsymbol{h}} = \left\{ (\boldsymbol{s}_i, \boldsymbol{s}_j) \in \mathcal{S}^2 \;\middle|\; \|\boldsymbol{s}_i - \boldsymbol{s}_j\| \in C_{\boldsymbol{h}} \right\}$.



Transformation and WLSE

$\Longrightarrow$

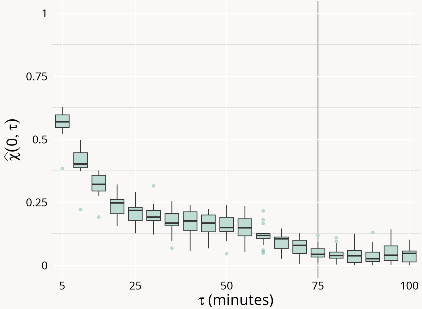


Spatial variogram $\widehat{\gamma}(\boldsymbol{h}, 0) = 2\widehat{\beta}_1 \|\boldsymbol{h}\|^{\widehat{\alpha}_1}$

Empirical temporal extremogram

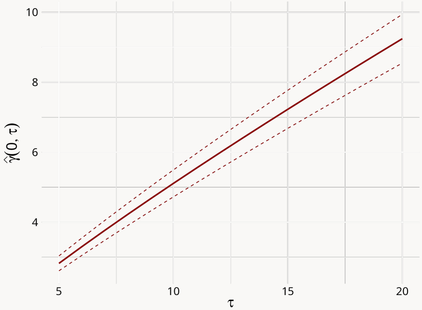
For a location $\boldsymbol{s} \in \mathcal{S}$, a high quantile q and $t_k \in \{t_1, \dots, t_T\}$,

$$\widehat{\chi}_q^{(\boldsymbol{s})}(\mathbf{0}, \tau) = \frac{\frac{1}{T-\tau} \sum_{k=1}^{T-\tau} \mathbb{1}_{\{X^*_{\boldsymbol{s},t_k} > q \,,\, X^*_{\boldsymbol{s},t_k+\tau} > q\}}}{\frac{1}{T} \sum_{k=1}^{T} \mathbb{1}_{\{X^*_{\boldsymbol{s},t_k} > q\}}}$$

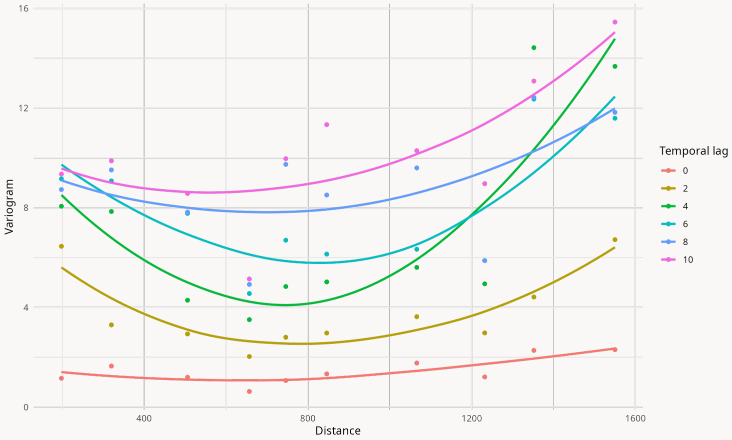


Transformation and WLSE

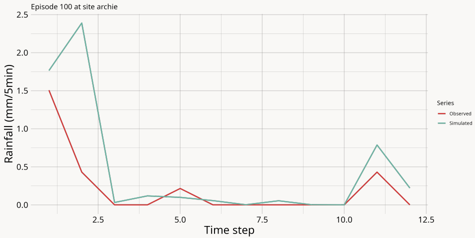
$\Longrightarrow$



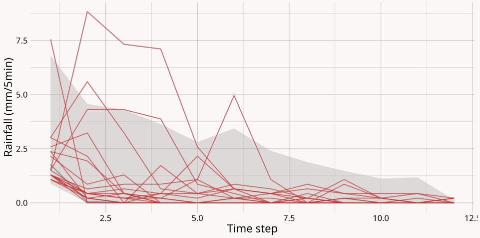
Temporal variogram $\widehat{\gamma}(\mathbf{0}, \tau) = 2\widehat{\beta}_2 |\tau|^{\widehat{\alpha}_2}$



Empirical spatio-temporal variogram $\gamma(\boldsymbol{h}, \tau)$ on OMSEV data



Simulation vs Observation - Episode 100



High advection intensity - South direction
95% CI over 1000 simulations vs 20 observations