

SPATIO-TEMPORAL MODELING OF URBAN EXTREME RAINFALL EVENTS AT HIGH RESOLUTION

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Chloé SERRE-COMBE ¹ Nicolas MEYER ¹ Thomas OPITZ ² Gwladys TOULEMONDE ¹

¹IMAG, Université de Montpellier, LEMON Inria

²INRAE, BioSP, Avignon



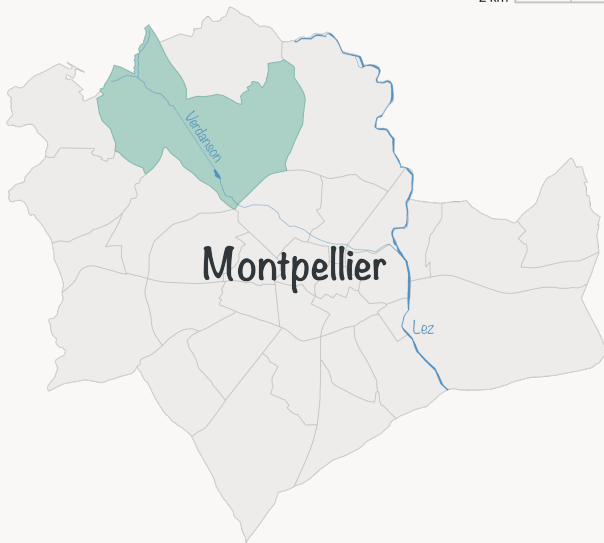


Floods in Montpellier, September 2022 and August 2015 (*Midi Libre*)

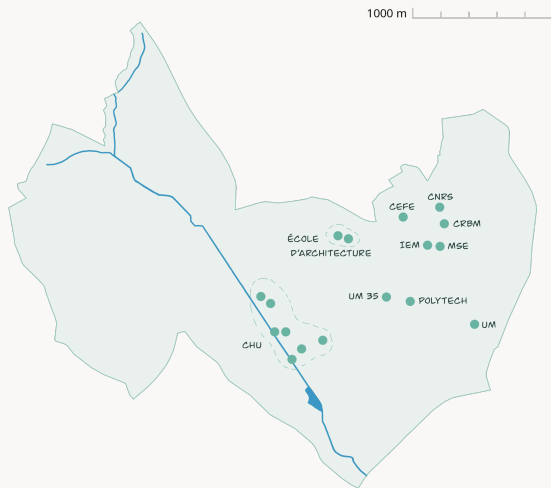
- ▶ Mediterranean events, localized rainfall
- ▶ Urban area, flood risks



2 km



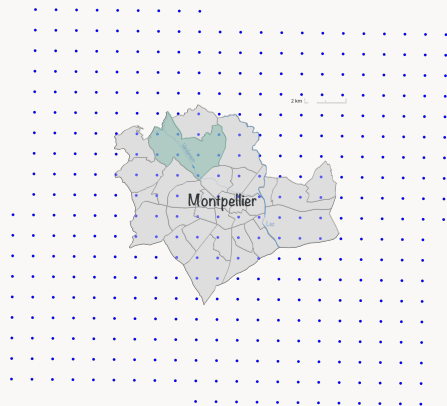
- North of Montpellier, *Hôpitaux-Facultés* district
- Verdanson water catchment, tributary of the Lez river



- **Source:** Urban observatory of HydroScience Montpellier (HSM)¹
- **Time period:** [2019, 2023[
- **Temp. resol.:** 5 minutes
- **Spatial resol.:** 77 m to 1531 m

¹FINAUD-GUYOT et al., 2023

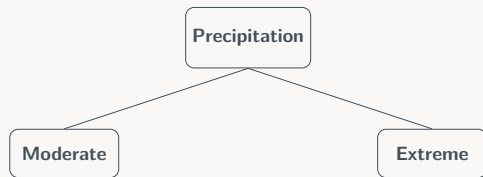
$$\mathcal{S} = \{17 \text{ rain gauges}\} \subset \mathbb{R}^2 \text{ and } \mathcal{T} \subset \mathbb{R}_+$$



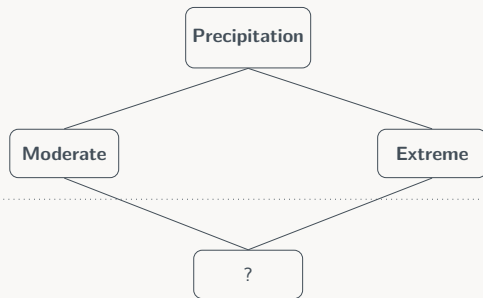
- **Source:** COMEPHORE, Météo France
- **Time period:** [1997, 2023[
- **Temporal resolution:** Every hour
- **Spatial resolution:** 1 km²

$$\mathcal{S} = \{400 \text{ pixels}\} \subset \mathbb{R}^2 \text{ and } \mathcal{T} \subset \mathbb{R}_+$$

More consistent data: HSM + COMEPHORE and Neural Network Downscaling.

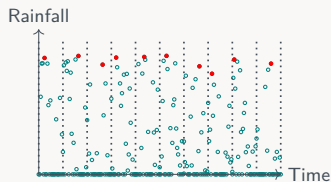
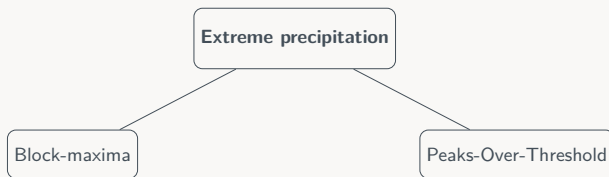


Univariate

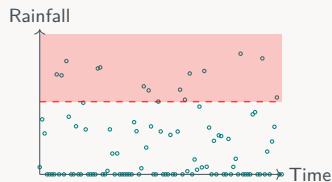


Univariate

EXTREME VALUE THEORY (EVT)



Generalized Extreme Value
distribution (GEV)



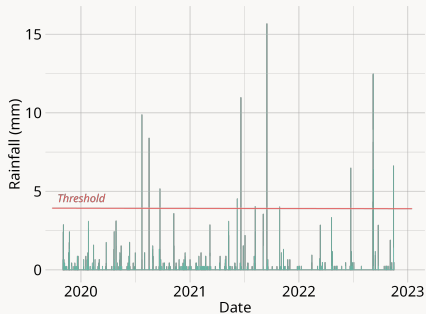
Generalized Pareto Distribution
(GPD)

Generalized Pareto Distribution

$$\overline{H}_\xi \left(\frac{x-u}{\sigma} \right) = \begin{cases} (1 + \xi \frac{x-u}{\sigma})_+^{-1/\xi} & \text{if } \xi \neq 0, \\ e^{-\frac{x-u}{\sigma}} & \text{if } \xi = 0, \end{cases}$$

where $a_+ = \max(a, 0)$, $\sigma > 0$, $x - u > 0$

- Models extreme precipitation
- Depends on a threshold choice



Polytech

Generalized Pareto Distribution



Extended GPD²

$$\overline{H}_\xi \left(\frac{x-u}{\sigma} \right) = \begin{cases} \left(1 + \xi \frac{x-u}{\sigma} \right)_+^{-1/\xi} & \text{if } \xi \neq 0, \\ e^{-\frac{x-u}{\sigma}} & \text{if } \xi = 0, \end{cases}$$

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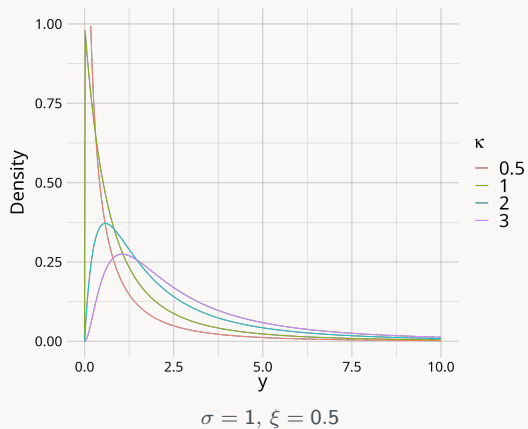
- Models extreme precipitation
- Depends on a threshold choice

$$F(x) = G \left(H_\xi \left(\frac{x}{\sigma} \right) \right),$$

where $G(x) = x^\kappa$, $\kappa > 0$

- Models **moderate and extreme** precipitation
- Avoids a threshold choice

²NAVEAU et al., 2016

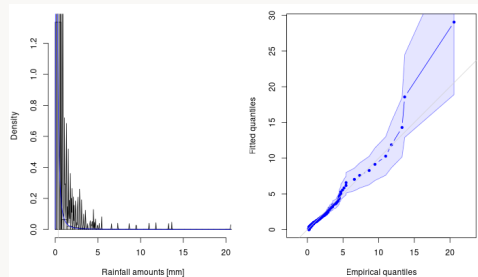
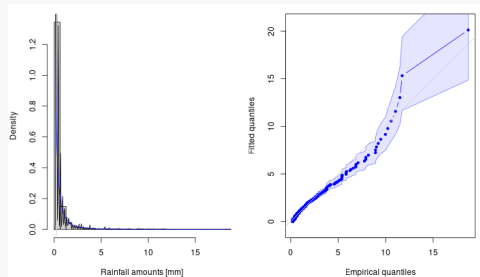


Extended GPD

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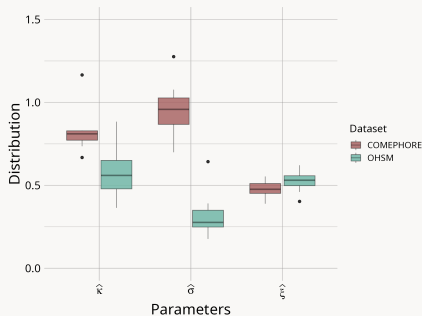
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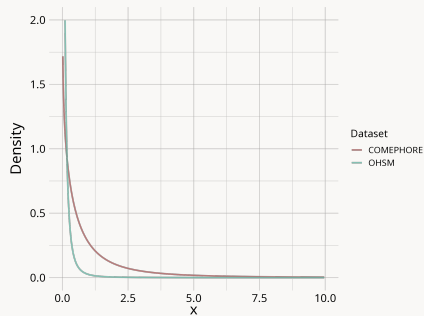


EGPD fitting for two rain gauges, CRBM (left) and CNRS (right) with left-censoring and 95% CI

Left-censoring: selected according to the RMSE criterion for each site individually³



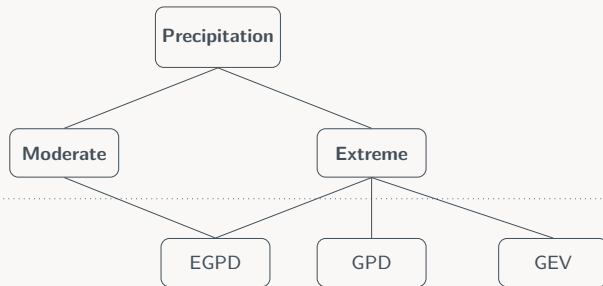
Parameter estimates



Density with mean parameters

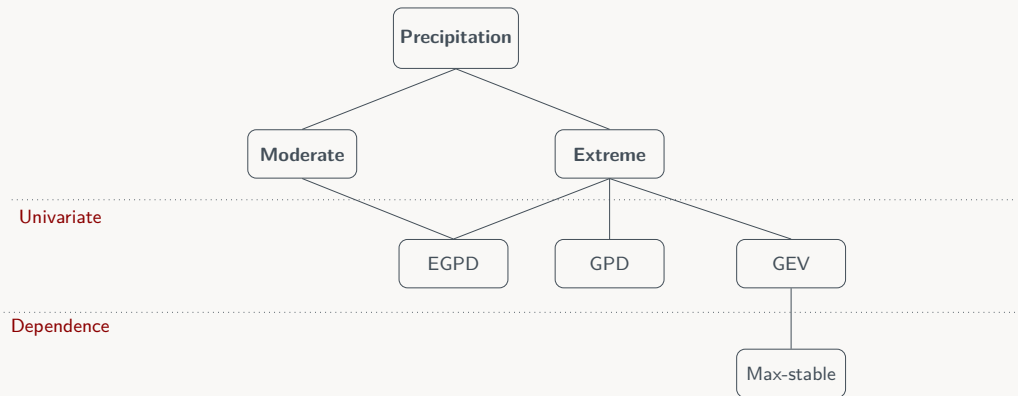
EGPD fitting for two rain gauges, with parameter estimates and density with mean parameters

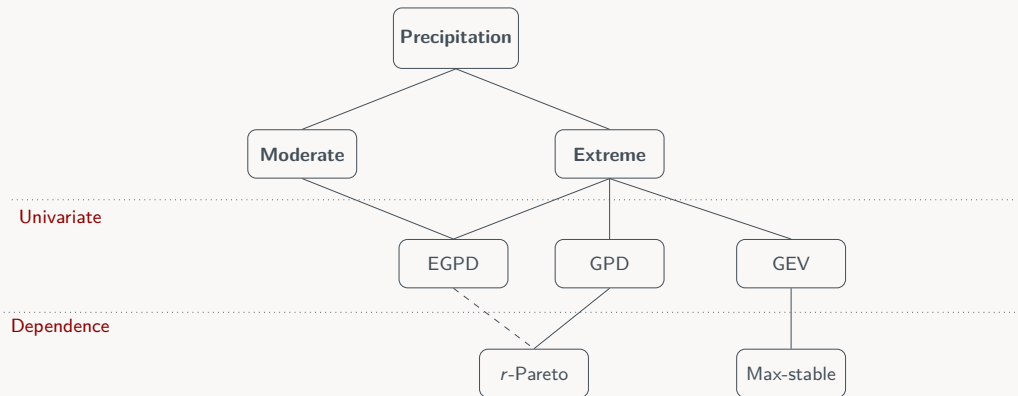
³HARUNA et al., 2023

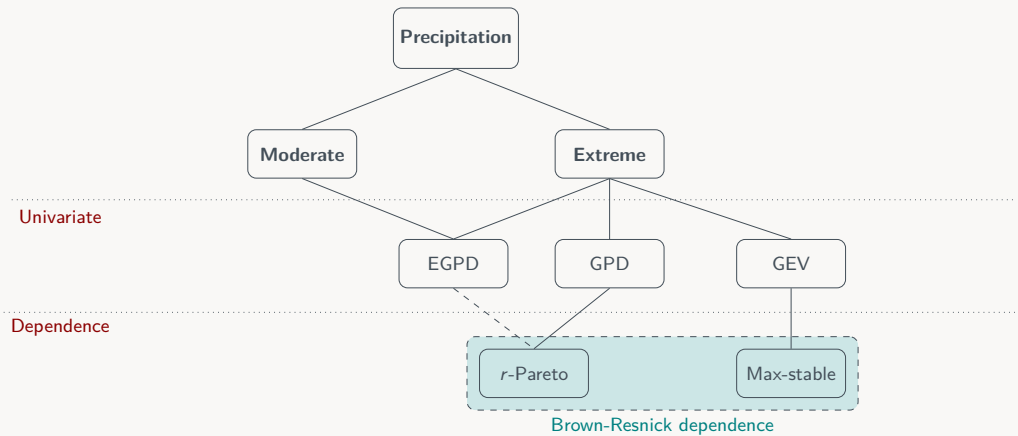


Univariate

Dependence







Rainfall random field: $\mathbf{X} = \{X_{s,t}, (s, t) \in \mathcal{S} \times \mathcal{T}\}$ stationary and isotropic

Let $\Lambda_{\mathcal{S}} \subset \mathbb{R}_+^2$ and $\Lambda_{\mathcal{T}} \subset \mathbb{R}_+$ be sets of spatial and temporal lags respectively.

Variogram

The spatio-temporal variogram of $\mathbf{X}$ is defined as

$$\gamma(\mathbf{h}, \tau) = \frac{1}{2} \text{Var}(X_{s,t} - X_{s+\mathbf{h}, t+\tau}), \mathbf{h} \in \Lambda_{\mathcal{S}}, \tau \in \Lambda_{\mathcal{T}}$$

- Describes the spatio-temporal variability
- Small $\gamma(\mathbf{h}, \tau)$: Strong dependence
- Large $\gamma(\mathbf{h}, \tau)$: Weaker dependence

Let $\Lambda_S \subset \mathbb{R}_+^2$ and $\Lambda_T \subset \mathbb{R}_+$ be sets of spatial and temporal lags respectively.

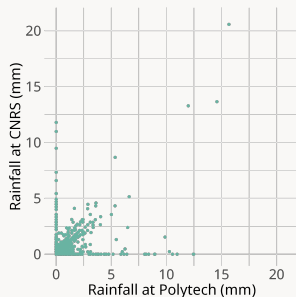
Spatio-temporal extremogram (COLES et al., 1999)

For all $\mathbf{h} \in \Lambda_S, \tau \in \Lambda_T$,

$$\chi(\mathbf{h}, \tau) = \lim_{q \rightarrow 1} \mathbb{P}(X_{s,t}^* > q \mid X_{s+\mathbf{h}, t+\tau}^* > q),$$

with $q \in [0, 1[$ and $X_{s,t}^*$ the uniform margins.

- ▶ $\chi(\mathbf{h}, \tau) \in [0, 1]$
- ▶ $\chi(\mathbf{h}, \tau) > 0$: asymptotic dependence
- ▶ $\chi(\mathbf{h}, \tau) = 0$: asymptotic independence



Let $\Lambda_S \subset \mathbb{R}_+^2$ and $\Lambda_T \subset \mathbb{R}_+$ be sets of spatial and temporal lags respectively.

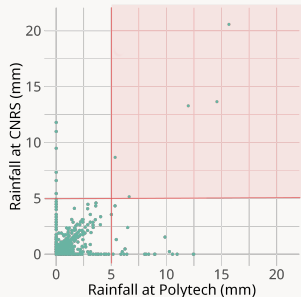
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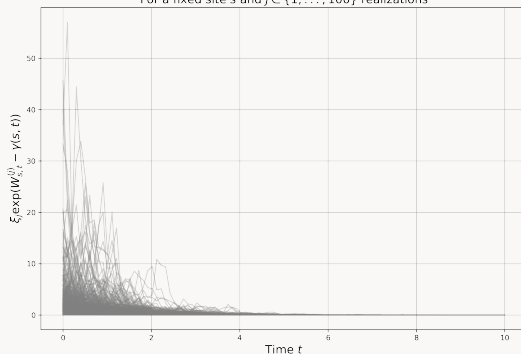
Spectral representation of a Brown-Resnick process (BROWN and RESNICK, 1977)

For all $s \in \mathcal{S}$ and $t \in \mathcal{T}$,

$$Z_{s,t} = \bigvee_{j=1}^{\infty} \xi_j e^{W_{s,t}^{(j)} - \gamma(s,t)}$$

- ▶ $(\xi_j)_{j \geq 1}$: points of a Poisson process with intensity $\xi^{-2} d\xi$
- ▶ $(W^{(j)})_{j \geq 1}$: independent replicates of an intrinsic stationary and isotropic Gaussian random field $\mathbf{W}$
- ▶ γ : spatio-temporal variogram of $\mathbf{W}$

For a fixed site s and $j \in \{1, \dots, 100\}$ realizations



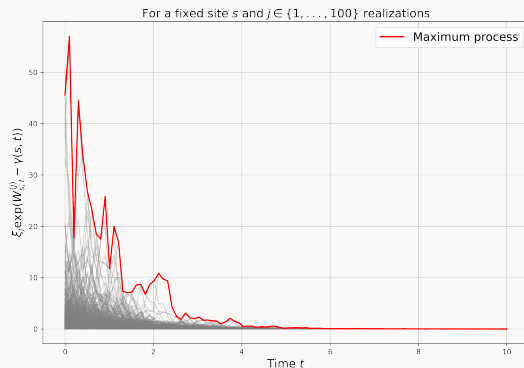
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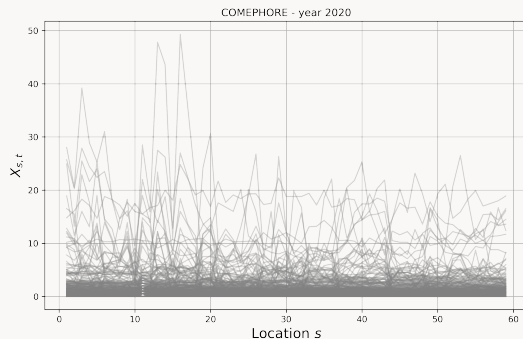


Definition (de FONDEVILLE and DAVISON, 2018)

For all $s \in \mathcal{S}$ and $t \in \mathcal{T}$, a risk function $r(\mathbf{X}) = X_{s_0, t_0}$,

$$X_{s,t} \mid r(\mathbf{X}) > u \stackrel{d}{\rightarrow} Y_{s,t} \quad \text{with} \quad Y_{s,t} = u R_{s,t} e^{W_{s,t} - W_{s_0, t_0} - \gamma(s - s_0, t - t_0)},$$

with (s_0, t_0) a given space-time location, u a high threshold and $R_{s,t} \sim \text{Pareto}(1)$.



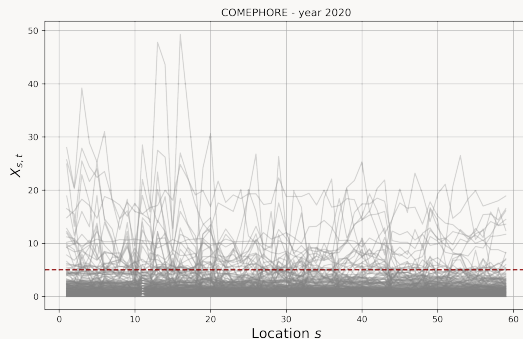
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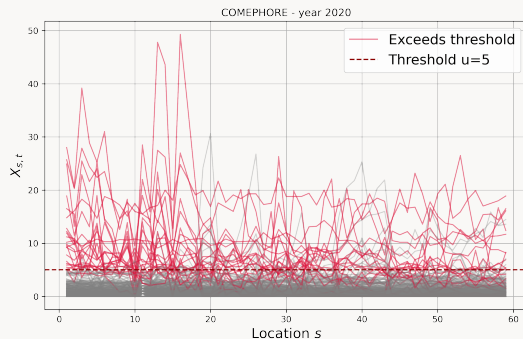
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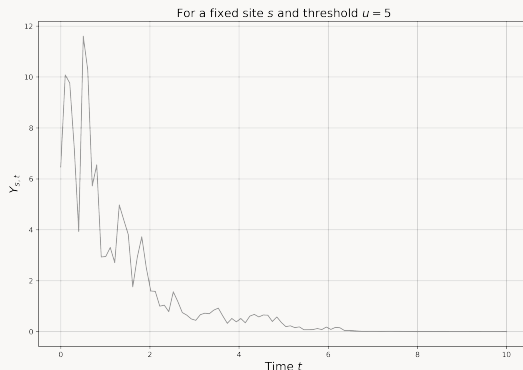
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with (s_0, t_0) a given space-time location, u a high threshold and $R_{s,t} \sim \text{Pareto}(1)$.



Let $\{X_{s,t} \mid r(\mathbf{X}) > u, s \in \mathcal{S}, t \in \mathcal{T}\}$ converges in distribution to a r -Pareto process with $r(\mathbf{X}) = X_{s_0, t_0}$.

Spatio-temporal r -extremogram

For all $\mathbf{h} \in \Lambda_{\mathcal{S}}, \tau \in \Lambda_{\mathcal{T}}$,

$$\begin{aligned}\chi_r(\mathbf{h}, \tau) &= \lim_{q \rightarrow 1} \mathbb{P}(X_{s_0 + \mathbf{h}, t_0 + \tau}^* > q \mid X_{s_0, t_0}^* > q) \\ &= \lim_{q \rightarrow 1} \mathbb{P}(X_{s_0 + \mathbf{h}, t_0 + \tau}^* > q),\end{aligned}$$

with $q \in [0, 1[$ corresponding to threshold u and $X_{s,t}^*$ the uniform margins.

Spatio-temporal r -variogram of W

For all $\mathbf{h} \in \Lambda_{\mathcal{S}}, \tau \in \Lambda_{\mathcal{T}}$,

$$\gamma_r(\mathbf{h}, \tau) = \frac{1}{2} \text{Var}(W_{s_0, t_0} - W_{s_0 + \mathbf{h}, t_0 + \tau}).$$

Let $\Lambda_S \subset \mathbb{R}_+^2$ and $\Lambda_T \subset \mathbb{R}_+$ be sets of spatial and temporal lags respectively.

Spatio-temporal extremogram with a Brown-Resnick dependence

Let $\mathbf{h} \in \Lambda_S$ and $\tau \in \Lambda_T$. We have

$$\chi(\mathbf{h}, \tau) = 2 \left(1 - \phi \left(\sqrt{\frac{1}{2} \gamma(\mathbf{h}, \tau)} \right) \right)$$

with ϕ the std normal c.d.f. and γ the variogram of $\mathbf{W}$.

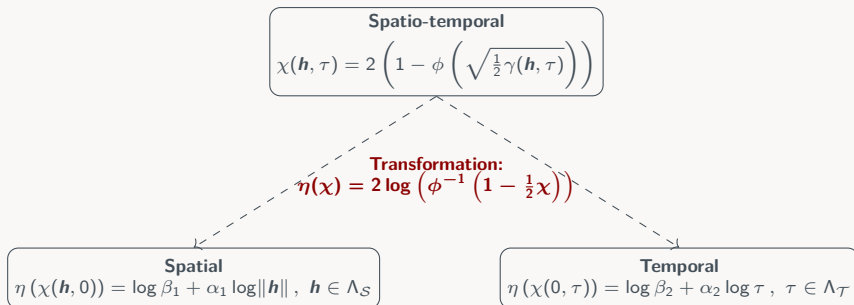
Dependence framework: BUHL et al., 2019

- ▶ Fractional Brownian motion
- ▶ Additive separability

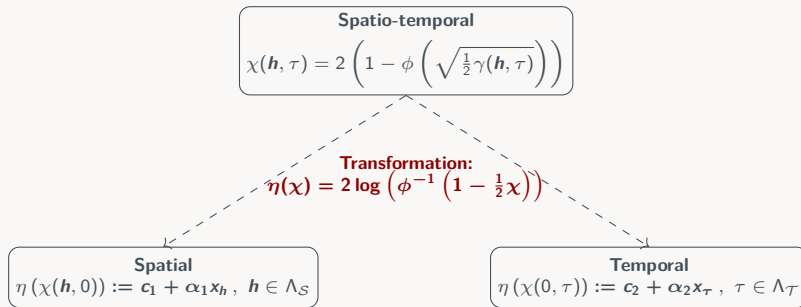
$$\frac{\gamma(\mathbf{h}, \tau)}{2} = \beta_1 \|\mathbf{h}\|^{\alpha_1} + \beta_2 |\tau|^{\alpha_2},$$

with $0 < \alpha_1, \alpha_2 \leq 2$, $\beta_1, \beta_2 > 0$.

Case of additive separability: $\frac{\gamma(h, \tau)}{2} = \beta_1 \|h\|^{\alpha_1} + \beta_2 |\tau|^{\alpha_2}, \quad 0 < \alpha_1, \alpha_2 \leq 2, \quad \beta_1, \beta_2 > 0$



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Weighted Least Squares Estimation (WLSE)

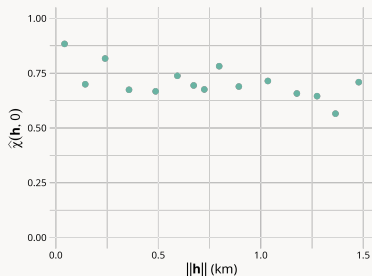
$$\begin{pmatrix} \hat{c}_i \\ \hat{\alpha}_i \end{pmatrix} = \operatorname{argmin}_{c_i, \alpha_i} \sum_x w_x \left(\eta(\hat{\chi}) - (c_i + \alpha_i x) \right)^2$$

Empirical spatial extremogram

For a fixed $t \in \mathcal{T}$ and q a high quantile,

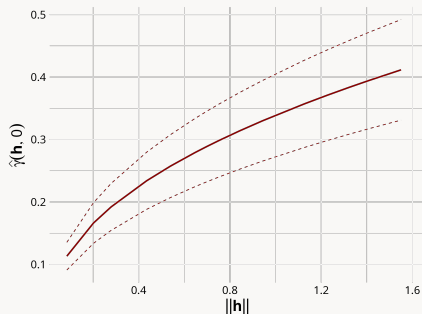
$$\hat{\chi}_q^{(t)}(\mathbf{h}, 0) = \frac{\frac{1}{|N_{\mathbf{h}}|} \sum_{i,j \mid (s_i, s_j) \in N_{\mathbf{h}}} \mathbb{1}_{\{X_{s_i, t}^* > q, X_{s_j, t}^* > q\}}}{\frac{1}{|S|} \sum_{i=1}^{|S|} \mathbb{1}_{\{X_{s_i, t}^* > q\}}},$$

where $C_{\mathbf{h}}$ are equifrequent distance classes and $N_{\mathbf{h}} = \{(s_i, s_j) \in S^2 \mid \|s_i - s_j\| \in C_{\mathbf{h}}\}$.



Transformation and WLSE

$\Rightarrow$

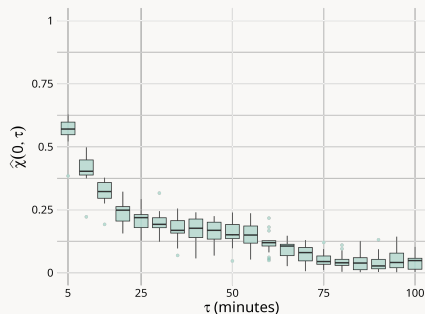


Spatial variogram $\hat{\gamma}(\mathbf{h}, 0) = 2\hat{\beta}_1 \|\mathbf{h}\|^{\hat{\alpha}_1}$

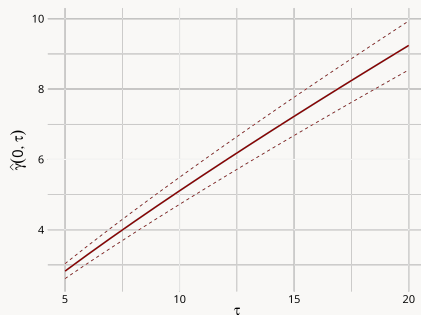
Empirical temporal extremogram

For a location $s \in S$, a high quantile q and $t_k \in \{t_1, \dots, t_T\}$,

$$\hat{\chi}_q^{(s)}(\mathbf{0}, \tau) = \frac{\frac{1}{T-\tau} \sum_{k=1}^{T-\tau} \mathbb{1}_{\{X_{s,t_k}^* > q, X_{s,t_k+\tau}^* > q\}}}{\frac{1}{T} \sum_{k=1}^T \mathbb{1}_{\{X_{s,t_k}^* > q\}}}$$



Transformation and WLSE



Temporal variogram $\hat{\gamma}(\mathbf{0}, \tau) = 2\hat{\beta}_2|\tau|^{\hat{\alpha}_2}$

Advection vector $\mathbf{V}$

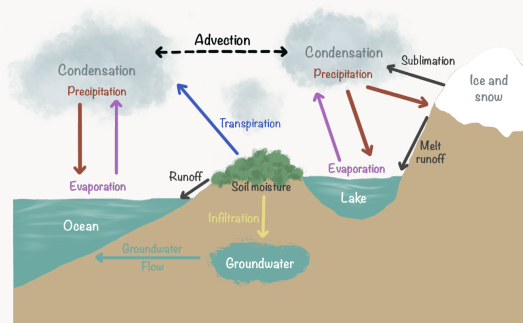
- Horizontal transport of air masses
- To relax the separability assumption

Lagrangian/Eulerian variogram

$$\gamma_L(\mathbf{h}, \tau) = \gamma(\mathbf{h} - \tau \mathbf{V}, \tau)$$

Dependence model

$$\frac{1}{2} \gamma_L(\mathbf{h}, \tau) = \beta_1 \|\mathbf{h} - \tau \mathbf{V}\|^{\alpha_1} + \beta_2 |\tau|^{\alpha_2}$$



Hydrologic cycle

Parameter optimization of $\Theta = (\beta_1, \beta_2, \alpha_1, \alpha_2, \mathbf{V})$

Joint excesses for r -Pareto processes:

$$\begin{aligned}
 K_{\mathbf{h}, \tau}(u) &= \sum_{s \in \mathcal{S}: (\mathbf{s}, \mathbf{s}_0) \in \mathcal{N}(\mathbf{h})} \sum_{t \in \mathcal{T}: t - t_0 = \tau} \mathbb{1}_{\{X_{\mathbf{s}, t} > u, X_{\mathbf{s}_0 + \mathbf{h}, t_0 + \tau} > u\}} \\
 &\sim \mathcal{B} \left(\sum_{(\mathbf{s}_0, t_0) \in \mathcal{R}} \#\{s \in \mathcal{S} : (\mathbf{s}, \mathbf{s}_0) \in \mathcal{N}(\mathbf{h})\}, \chi_{r, \Theta}(\mathbf{h}, \tau) \right), \text{ for a large } u
 \end{aligned}$$

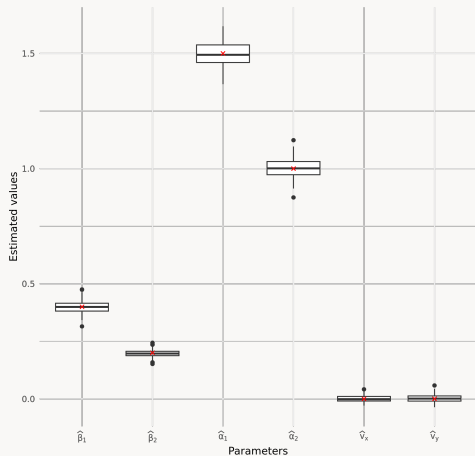
with $\mathcal{R}$ a specific set of r -conditioning spatio-temporal locations.

Composite log-likelihood:

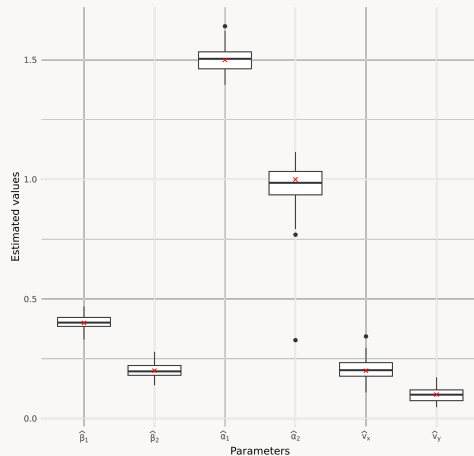
$$l_C(\Theta) \propto \sum_{(\mathbf{s}_0, t_0) \in \mathcal{R}} \sum_{(\mathbf{s}, \mathbf{s}_0) \in \mathcal{N}(\mathbf{h})} \sum_{t \in \mathcal{T}: t - t_0 = \tau} k_{\mathbf{s}, t} \log \chi_{r, \Theta}(\mathbf{h}, \tau) + (1 - k_{\mathbf{s}, t}) \log(1 - \chi_{r, \Theta}(\mathbf{h}, \tau)).$$

where $\chi_{r, \Theta}(\mathbf{h}, \tau)$ is the r -extremogram.

VALIDATION ON SIMULATIONS



Without advection



With advection

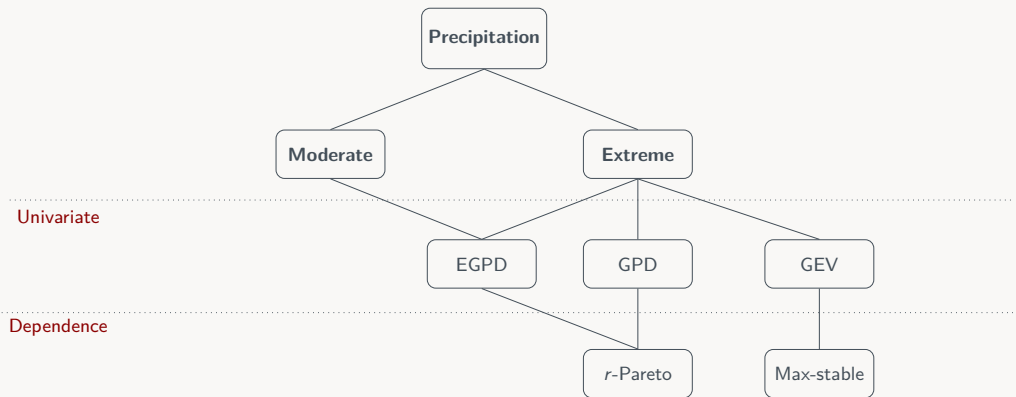
Parameter estimation on 100 simulations of 1000 replicates of r -Pareto processes with 25 sites and 30 time observations

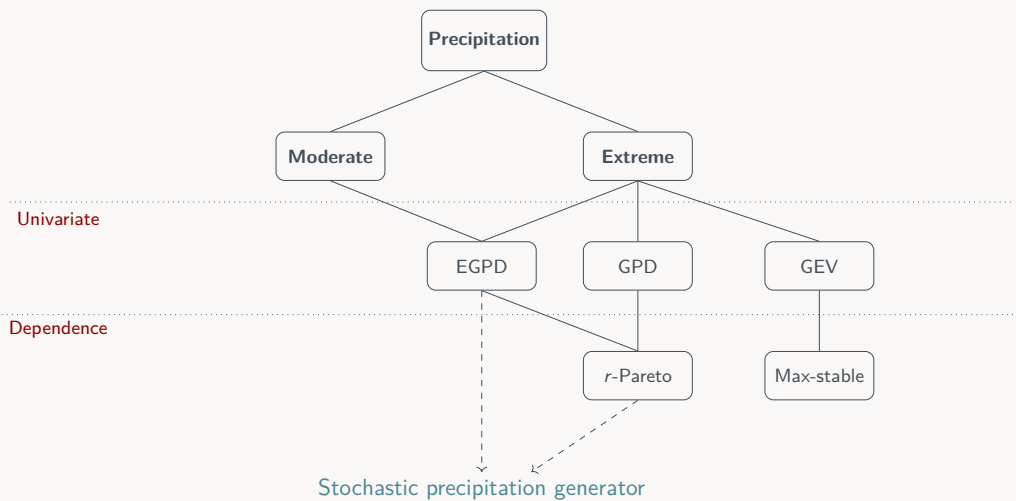
Estimate constant advection $\hat{\mathbf{V}}$

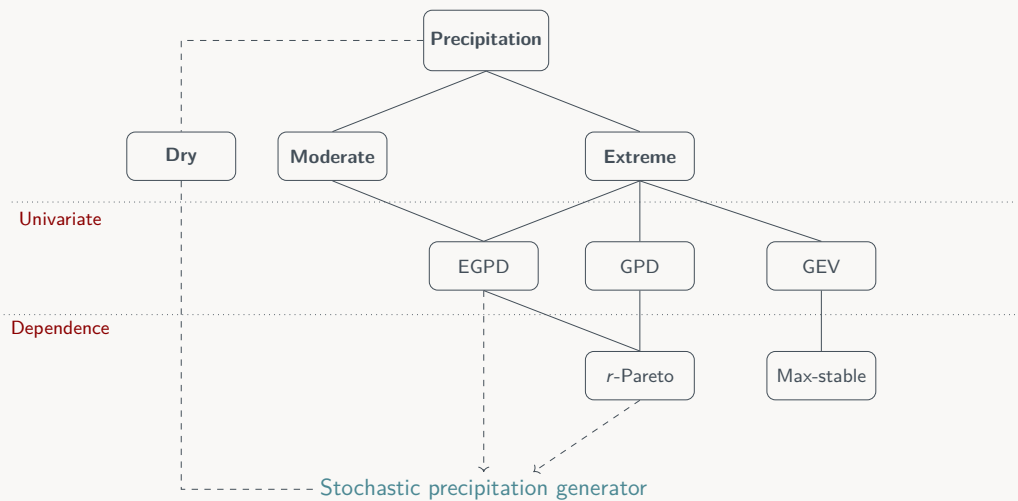
- ▶ Use **COMEPHORE** data.
- ▶ Initial parameters $\beta_1, \beta_2, \alpha_1, \alpha_2$: WLSE
- ▶ Initial advection: wind data

Estimate variogram parameters on HSM data

- ▶ Initial parameters $\beta_1, \beta_2, \alpha_1, \alpha_2$: WLSE
- ▶ Initial advection: $\hat{\mathbf{V}}$ (from COMEPHORE)

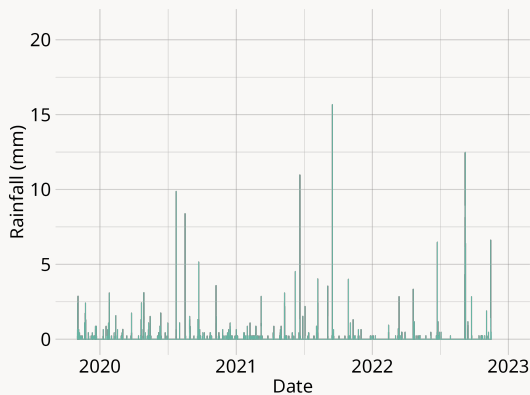
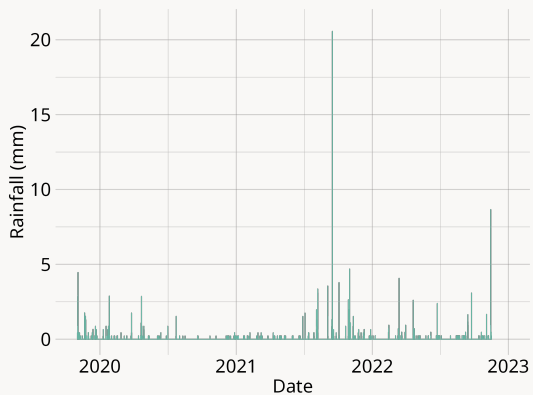




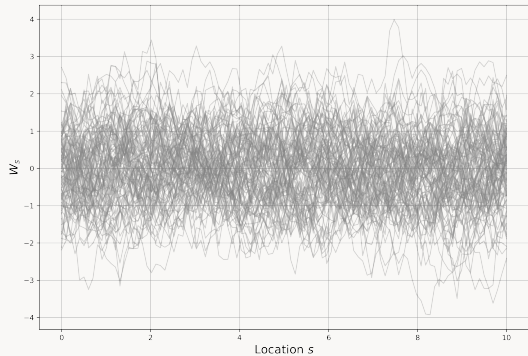


Thank you for your attention!

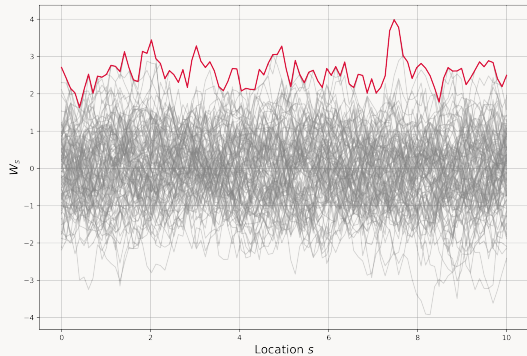
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Rainfall amounts on CNRS and Polytech rain gauges



100 Gaussian processes



Maximum process in red