

SPATIO-TEMPORAL MODELING OF URBAN EXTREME RAINFALL EVENTS AT HIGH RESOLUTION

Nice, February 10, 2026

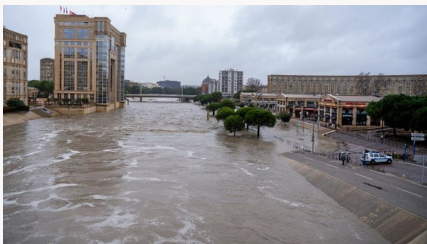
Chloé SERRE-COMBE ¹ Nicolas MEYER ¹ Thomas OPITZ ² Gwladys TOULEMONDE ¹

¹IMAG, Université de Montpellier, LEMON Inria

²INRAE, BioSP, Avignon



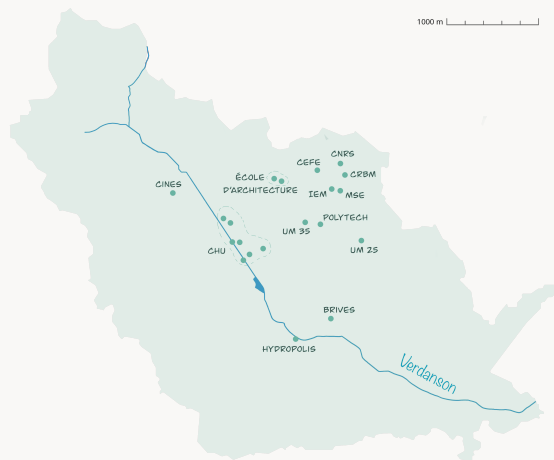
GENERAL CONTEXT - MONTPELLIER, FRANCE



Floods in Montpellier, August 2015 and December 2025 (*Midi Libre*)

- ▶ Mediterranean events, localized rainfall
- ▶ Urban area, flood risks



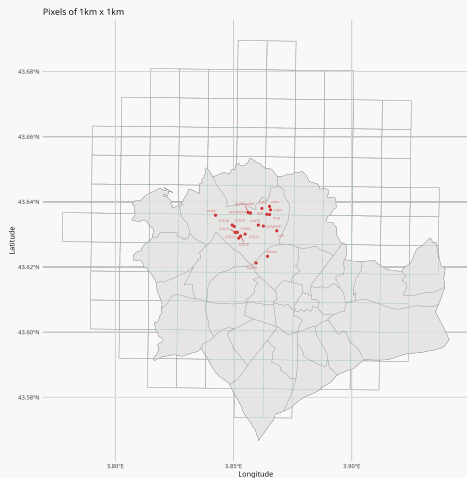


- ▶ **Study area:** Verdanson water catchment
- ▶ **Source:** Urban observatory of HydroScience Montpellier (HSM)²
- ▶ **Time period:** [Sept.2019, Jan.2025[
- ▶ **Temporal resolution:** 5 minutes
- ▶ **Spatial resolution:** 77 m to 2259 m

¹Observ. Montpellierain et au Sud de l'Eau dans la Ville

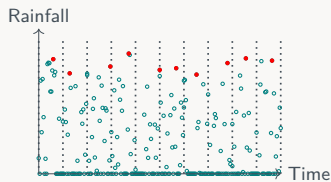
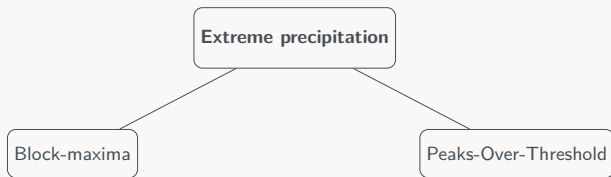
²data2023pluvio

$$\mathcal{S} = \{20 \text{ rain gauges}\} \subset \mathbb{R}^2 \text{ and } \mathcal{T} \subset \mathbb{R}_+$$

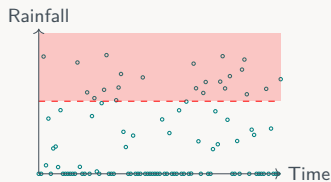


- ▶ **Source:** COMEPHORE, Météo France
- ▶ **Time period:** [1997, 2023[
- ▶ **Temporal resolution:** Every hour
- ▶ **Spatial resolution:** 1 km²

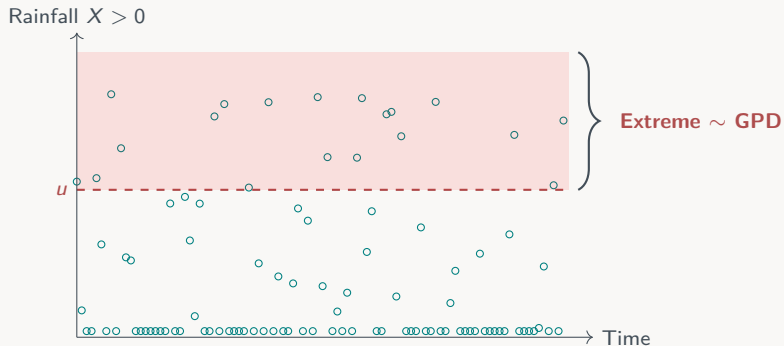
EXTREME VALUE THEORY (EVT)



Generalized Extreme Value distribution (GEV)



Generalized Pareto Distribution (GPD)

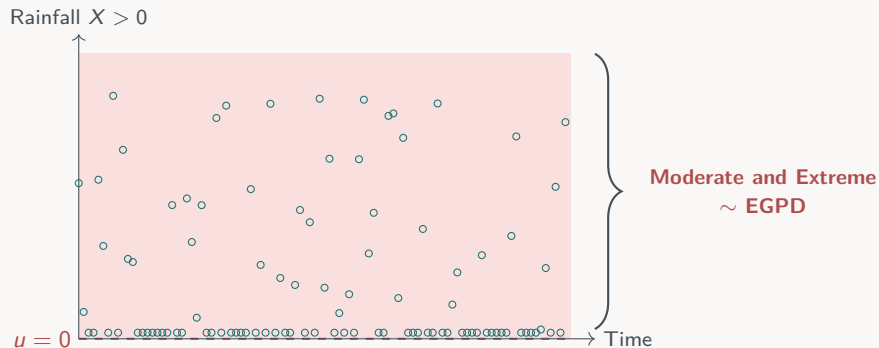


► Generalized Pareto Distribution (GPD)¹

$$X \mid X > u \sim \underbrace{H_{\xi}}_{\text{GPD}(\xi, \sigma, u)} \text{ with } \xi \in \mathbb{R}, \sigma > 0.$$

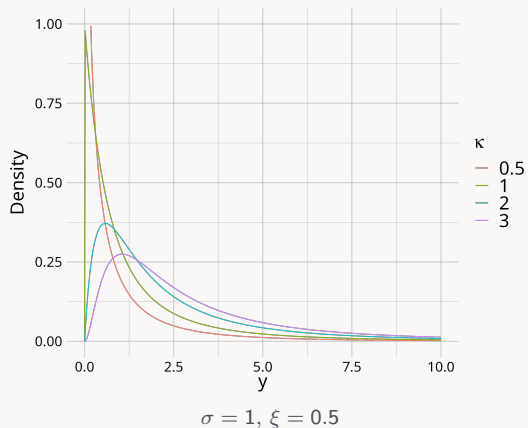
ξ : shape (tail behavior), σ : scale, u : threshold

¹pickands1975statistical



► **Extended Generalized Pareto Distribution (EGPD)²**

$$X \sim \underbrace{G(H_\xi)}_{\text{EGPD}(\xi, \sigma, \kappa)} \text{ with } G(x) = x^\kappa, \kappa > 0$$

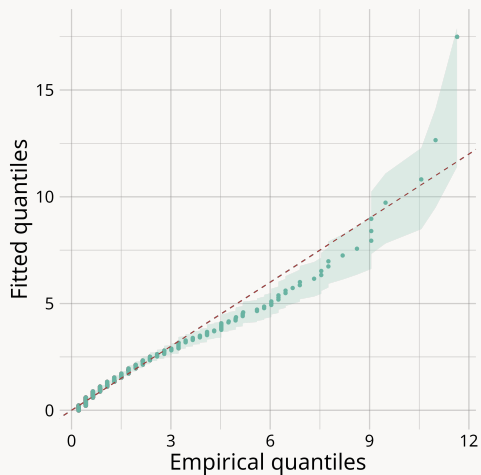
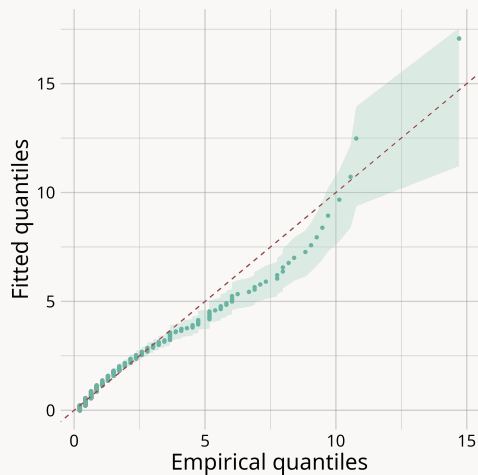


Extended Generalized Pareto Distribution

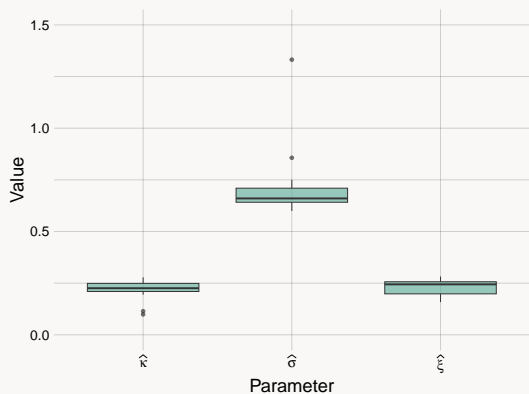
$$F(x) = G \left(H_{\xi} \left(\frac{x}{\sigma} \right) \right),$$

where $G(x) = x^{\kappa}, \kappa > 0$

- ▶ κ : controls the bulk of the distribution
- ▶ Models **moderate and extreme** precipitation
- ▶ Avoids a threshold choice

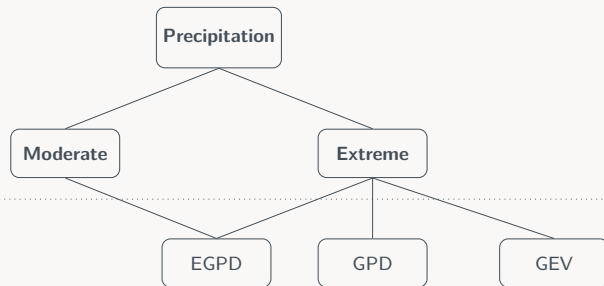


EGPD fitting for two rain gauges, CRBM (left) and UM (right) with left-censoring and 95% CI



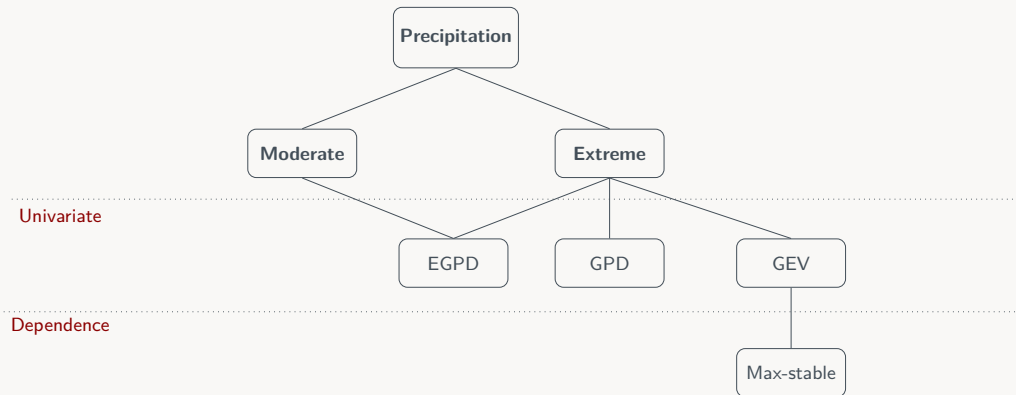
Estimated parameters across rain gauges

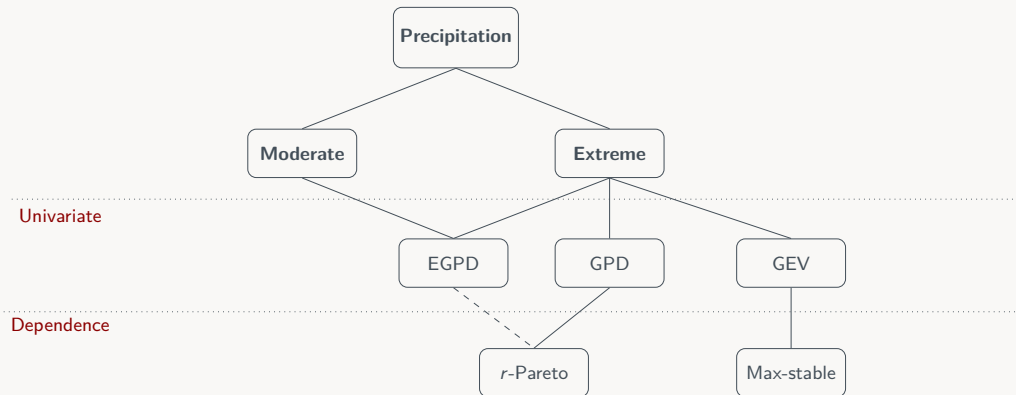
- ▶ $\hat{\xi} \approx 0.25$: extreme rainfall tail
- ▶ $\hat{\kappa} \approx 0.25$: convective rainfall
- ▶ $\hat{\sigma} \approx 0.6$: short-duration scale

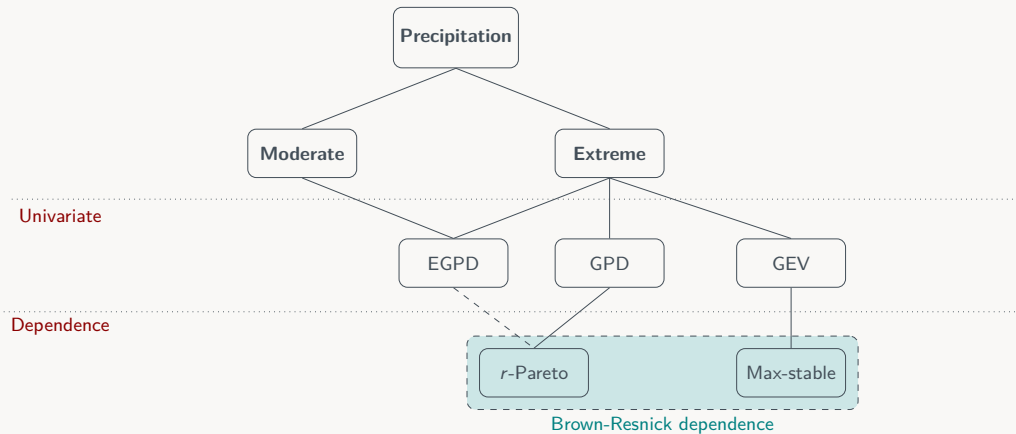


Univariate

Dependence







Rainfall random field: $\mathbf{X} = \{X_{s,t}, (s, t) \in \mathcal{S} \times \mathcal{T}\}$

Let $\Lambda_{\mathcal{S}} \subset \mathbb{R}_+^2$ and $\Lambda_{\mathcal{T}} \subset \mathbb{R}_+$ be sets of spatial and temporal lags respectively.

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Variogram (matheron1963principles)

$$\gamma(\mathbf{h}, \tau) = \frac{1}{2} \text{Var}(X_{s,t} - X_{s+\mathbf{h}, t+\tau})$$

- Quantifies variability
- Higher $\gamma(\mathbf{h}, \tau) \rightarrow$ weaker dependence

$$\mathbf{h} \in \Lambda_{\mathcal{S}}, \tau \in \Lambda_{\mathcal{T}}$$

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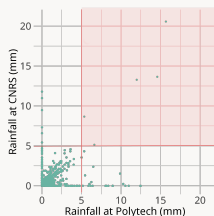
- Quantifies variability
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$\mathbf{h} \in \Lambda_{\mathcal{S}}, \tau \in \Lambda_{\mathcal{T}}, X_{s,t}^*$ uniform margins.

Extremogram (davis2009extremogram)

$$\chi(\mathbf{h}, \tau) = \lim_{q \rightarrow 1} \mathbb{P}(X_{s,t}^* > q \mid X_{s+\mathbf{h}, t+\tau}^* > q)$$

- Measures tail dependence
- Higher $\chi(\mathbf{h}, \tau) \rightarrow$ stronger dependence

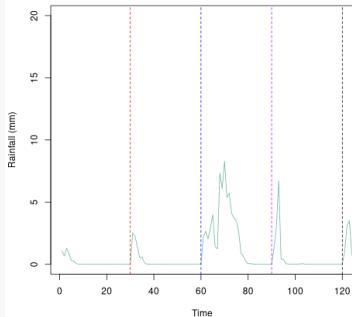


Definition (de_fondeville_high-dimensional_2018)

For all $s \in \mathcal{S}$ and $t \in \mathcal{T}$, a risk function $r(\mathbf{X}) = X_{s_0, t_0}$,

$$u^{-1}X_{s,t} \mid X_{s_0, t_0} > u \xrightarrow{d} Y_{s,t} \quad \text{with} \quad Y_{s,t} = Re^{W_{s,t} - W_{s_0, t_0} - \gamma(s-s_0, t-t_0)},$$

where (s_0, t_0) is a space-time location, u is a high threshold, $R \sim \text{Pareto}(1)$, $W_{s,t}$ is a Gaussian process.



► r -extremogram

$$\chi_r(\mathbf{h}, \tau) = \lim_{q \rightarrow 1} \mathbb{P}(X_{s_0+h, t_0+\tau}^* > q \mid X_{s_0, t_0}^* > q)$$

► Brown-Resnick dependence structure

$$\chi_r(\mathbf{h}, \tau) = 2 \left(1 - \phi \left(\sqrt{\frac{1}{2} \gamma(\mathbf{h}, \tau)} \right) \right)$$

Spatio-temporal extremogram with a Brown-Resnick dependence

Let $\mathbf{h} \in \Lambda_S$ and $\tau \in \Lambda_T$. We have

$$\chi(\mathbf{h}, \tau) = 2 \left(1 - \phi \left(\sqrt{\frac{1}{2} \gamma(\mathbf{h}, \tau)} \right) \right)$$

with ϕ the std normal c.d.f. and γ the variogram of \mathbf{W} .

Spatio-temporal extremogram with a Brown-Resnick dependence

Let $\mathbf{h} \in \Lambda_S$ and $\tau \in \Lambda_T$. We have

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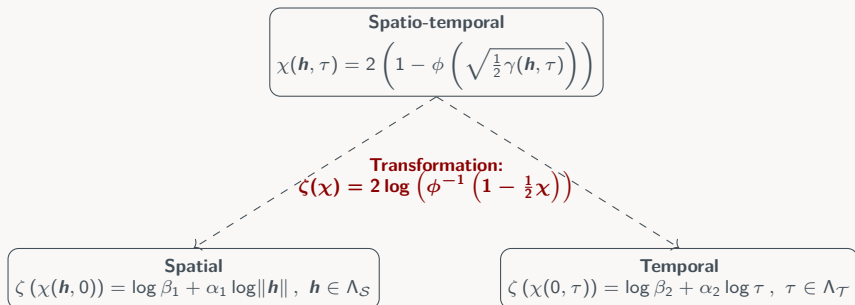
with ϕ the std normal c.d.f. and γ the variogram of \mathbf{W} .

Separable model: Fractional Brownian motion with additive separability.

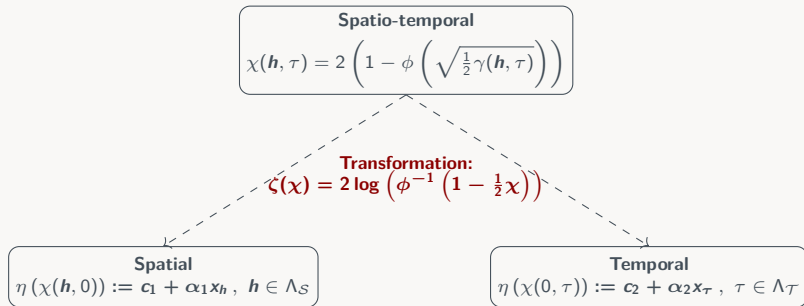
$$\frac{\gamma(\mathbf{h}, \tau)}{2} = \beta_1 \|\mathbf{h}\|^{\alpha_1} + \beta_2 |\tau|^{\alpha_2}$$

with $0 < \alpha_1, \alpha_2 \leq 2$, $\beta_1, \beta_2 > 0$ (**buhl2019semiparametric**).

Case of additive separability: $\frac{\gamma(\mathbf{h}, \tau)}{2} = \beta_1 \|\mathbf{h}\|^{\alpha_1} + \beta_2 |\tau|^{\alpha_2}$, $0 < \alpha_1, \alpha_2 \leq 2$, $\beta_1, \beta_2 > 0$



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Weighted Least Squares Estimation (WLSE)

$$\begin{pmatrix} \hat{c}_i \\ \hat{\alpha}_j \end{pmatrix} = \operatorname{argmin}_{c_i, \alpha_j} \sum_x w_x (\zeta(\hat{x}) - (c_i + \alpha_j x))^2$$

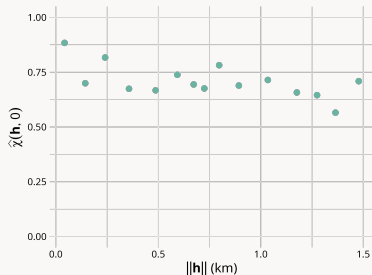
Empirical spatial extremogram

For a fixed $t \in \mathcal{T}$ and q a high quantile,

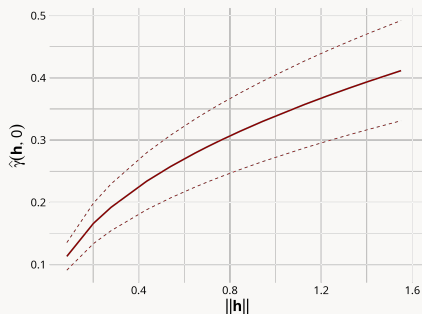
$$\hat{\chi}_q^{(t)}(\mathbf{h}, 0) = \frac{\frac{1}{|N_{\mathbf{h}}|} \sum_{i,j | (s_i, s_j) \in N_{\mathbf{h}}} \mathbb{1}_{\{X_{s_i, t}^* > q, X_{s_j, t}^* > q\}}}{\frac{1}{|S|} \sum_{i=1}^{|S|} \mathbb{1}_{\{X_{s_i, t}^* > q\}}},$$

where C_h are equifrequent distance classes and

$$N_{\mathbf{h}} = \left\{ (s_i, s_j) \in S^2 \mid \|s_i - s_j\| \in C_h \right\}.$$



Transformation and WLSE

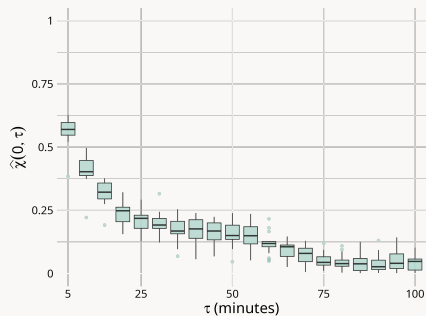


Spatial variogram $\hat{\gamma}(\mathbf{h}, 0) = 2\hat{\beta}_1 \|\mathbf{h}\|^{\hat{\alpha}_1}$

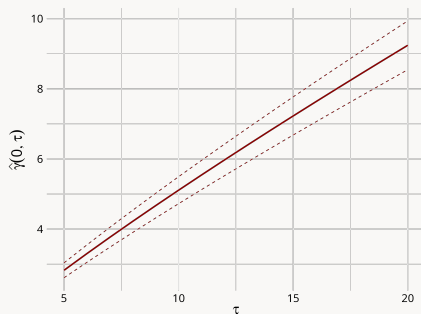
Empirical temporal extremogram

For a location $s \in \mathcal{S}$, a high quantile q and $t_k \in \{t_1, \dots, t_T\}$,

$$\hat{\chi}_q^{(s)}(\mathbf{0}, \tau) = \frac{\frac{1}{T-\tau} \sum_{k=1}^{T-\tau} \mathbb{1}\{X_{s,t_k}^* > q, X_{s,t_k+\tau}^* > q\}}{\frac{1}{T} \sum_{k=1}^T \mathbb{1}\{X_{s,t_k}^* > q\}}$$

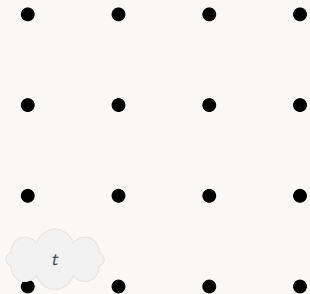


Transformation and WLSE

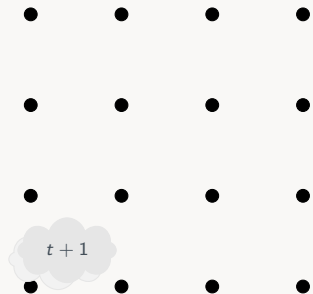


Temporal variogram $\hat{\gamma}(\mathbf{0}, \tau) = 2\hat{\beta}_2|\tau|^{\hat{\alpha}_2}$

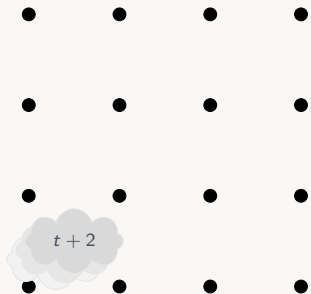
Separable model



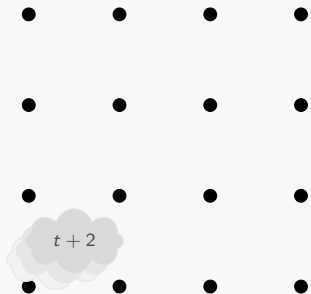
Separable model



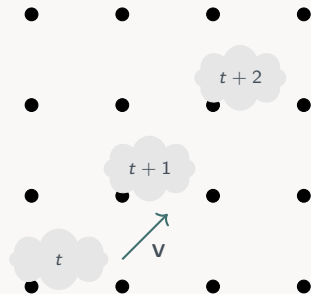
Separable model



Separable model



Non-separable model



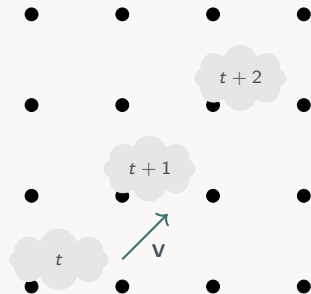
Towards more realistic modeling: introduce advection \mathbf{V} to relax separability

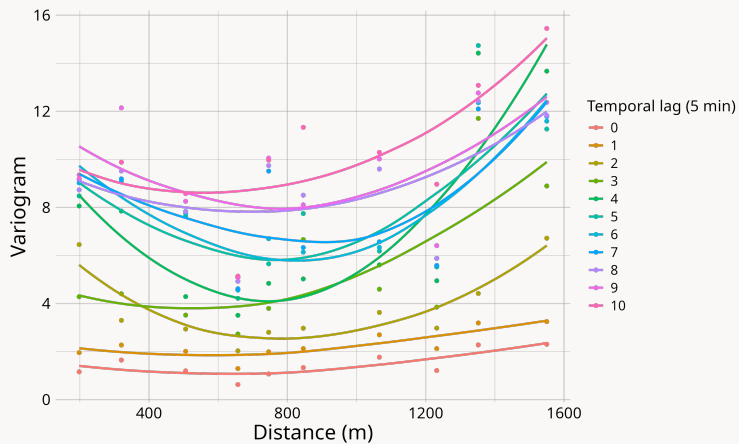
$$\gamma_L(\mathbf{h}, \tau) = \gamma(\mathbf{h} - \tau \mathbf{V}, \tau)$$

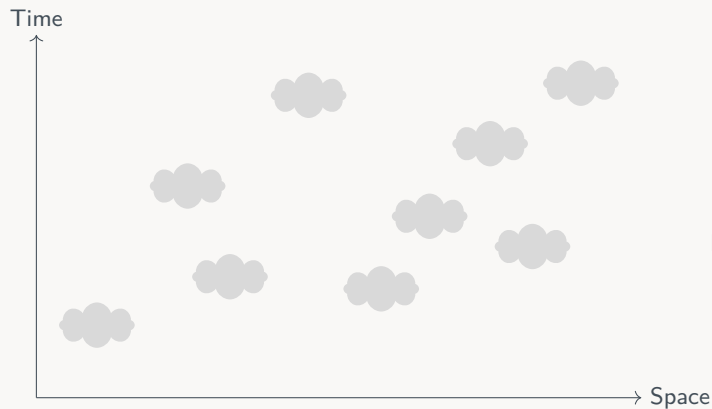
$$\Rightarrow \frac{1}{2} \gamma_L(\mathbf{h}, \tau) = \beta_1 \|\mathbf{h} - \tau \mathbf{V}\|^{\alpha_1} + \beta_2 |\tau|^{\alpha_2}$$

► Parameters: $\Theta = (\beta_1, \beta_2, \alpha_1, \alpha_2, \mathbf{V})$

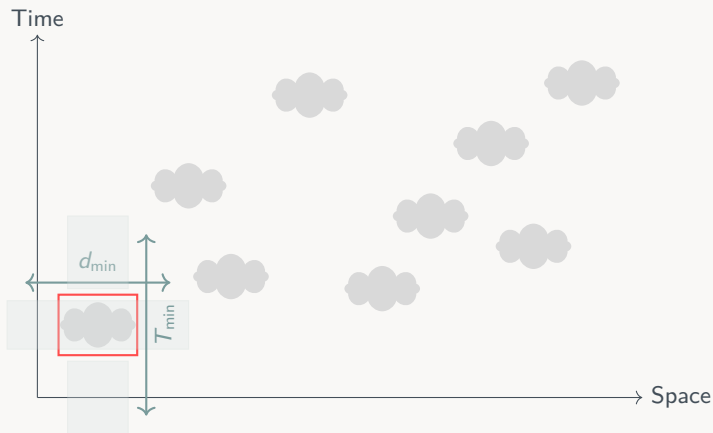
Non-separable model



Empirical spatio-temporal variogram $\gamma(\mathbf{h}, \tau)$ on OMSEV data



Extreme episodes:
Each episode is characterized
by (s_0, t_0) for which $X_{s_0, t_0} > u$

**Episode selection:**

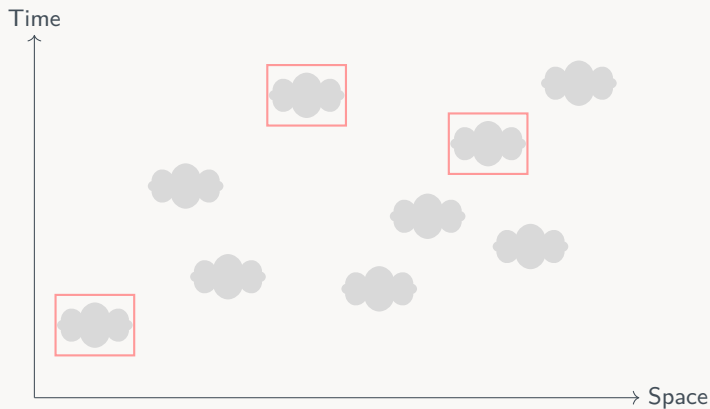
Only episodes separated by

- spatial distance $\geq d_{\min}$

OR

- temporal gap $\geq T_{\min}$

\Rightarrow reduces dependence
between **selected episodes** $\in \mathcal{E}$.

**Episode selection:**

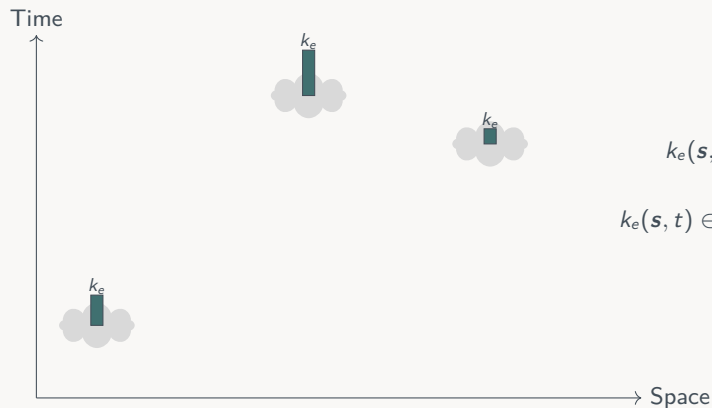
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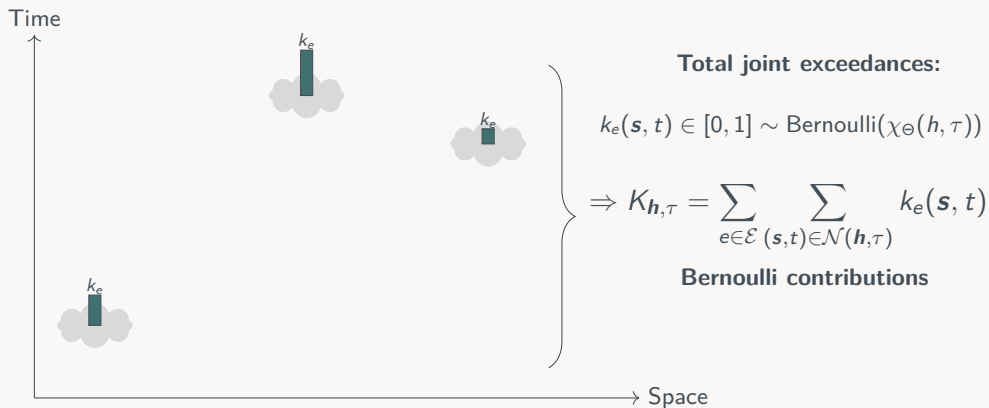
Joint exceedances

$$k_e(\mathbf{s}, t) = \mathbb{1}_{\{X_{s_0, t_0} > u, X_{\mathbf{s}, t} > u\}},$$

with $(\mathbf{s}, t) \in \mathcal{N}(\mathbf{h}, \tau)$

$$k_e(\mathbf{s}, t) \in [0, 1] \sim \text{Bernoulli}(\chi_{\Theta}(\mathbf{h}, \tau))$$

$$\text{with } \mathcal{N}(\mathbf{h}, \tau) = \{(\mathbf{s}, t) \in \mathcal{S} \times \mathcal{T} \mid \mathbf{s} - \mathbf{s}_0 = \mathbf{h}, t - t_0 = \tau\}$$



$$\text{with } \mathcal{N}(\mathbf{h}, \tau) = \{(\mathbf{s}, t) \in \mathcal{S} \times \mathcal{T} \mid \mathbf{s} - \mathbf{s}_0 = \mathbf{h}, t - t_0 = \tau\}$$

Bernoulli contributions (large u):

$$k_e(\mathbf{s}, t) = \mathbf{1}_{\{X_{s_0, t_0} > u, X_{\mathbf{s}, t} > u\}}, \quad (\mathbf{s}, t) \in \mathcal{N}(\mathbf{h}, \tau),$$

each treated as

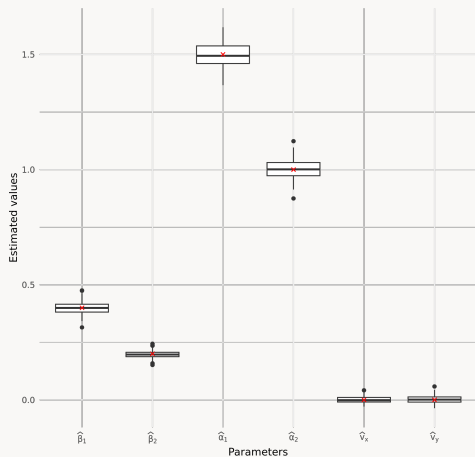
$$k_e(\mathbf{s}, t) \sim \text{Bernoulli}(\chi_{\Theta}(\mathbf{h}, \tau)).$$

Composite log-likelihood

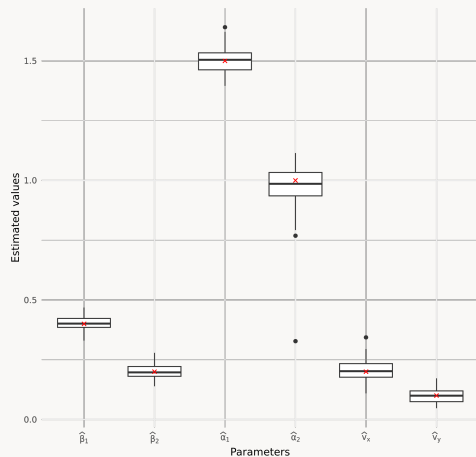
$$l_C(\Theta) \propto \sum_{e \in \mathcal{E}} \sum_{(\mathbf{h}, \tau) \in \Lambda_S \times \Lambda_T} \sum_{(\mathbf{s}, t) \in \mathcal{N}(\mathbf{h}, \tau)} k_e(\mathbf{s}, t) \log \chi_{\Theta}(\mathbf{h}, \tau) + (1 - k_e(\mathbf{s}, t)) \log(1 - \chi_{\Theta}(\mathbf{h}, \tau)).$$

- ▶ Optimization via maximization of $l_C(\Theta)$
- ▶ Initial parameters: from WLSE of the separable model (i.e., $\mathbf{V} = \mathbf{0}$)

VALIDATION: CONSTANT ADVECTION



Without advection



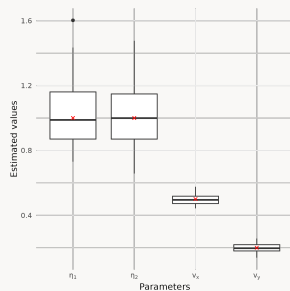
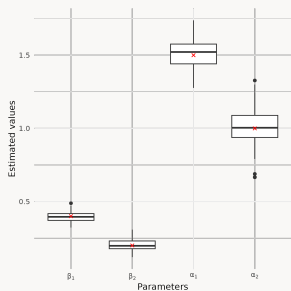
With advection

50 simulations, 500 replicates, 25 sites, 30 time steps

EPISODE-WISE ADVECTION TO GLOBAL MODEL

$$\mathbf{v}^{\text{emp}} = \begin{bmatrix} \mathbf{v}^{(1)} \\ \mathbf{v}^{(2)} \\ \vdots \\ \mathbf{v}^{(N)} \end{bmatrix} \in \mathbb{R}^{N \times 2} \xrightarrow{\text{global model}} \mathbf{v}^{\text{final}} = \underbrace{\eta_1 \|\mathbf{v}^{\text{emp}}\|^{\eta_2} \frac{\mathbf{v}^{\text{emp}}}{\|\mathbf{v}^{\text{emp}}\|}}_{\text{unified transformation} \Rightarrow \eta \text{ to estimate}}, \eta_1 > 0, \eta_2 > 0$$

one component = one episode



50 simulations, 500 replicates, 25 sites, 30 time steps

$$\mathbf{v}^{\text{emp}} = \begin{bmatrix} \mathbf{v}^{(1)} \\ \mathbf{v}^{(2)} \\ \vdots \\ \mathbf{v}^{(N)} \end{bmatrix} \in \mathbb{R}^{N \times 2} \xrightarrow{\text{global model}} \mathbf{v}^{\text{final}} = \underbrace{\eta_1 \|\mathbf{v}^{\text{emp}}\|^{\eta_2} \frac{\mathbf{v}^{\text{emp}}}{\|\mathbf{v}^{\text{emp}}\|}}_{\text{unified transformation} \Rightarrow \eta \text{ to estimate}}, \eta_1 > 0, \eta_2 > 0$$

one component = one episode

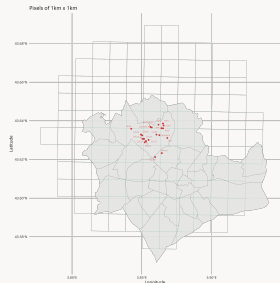
Parameters: $\Theta = (\beta_1, \beta_2, \alpha_1, \alpha_2, \eta_1, \eta_2)$

Challenge:

OMSEV data \Rightarrow limited information for advection

Approach:

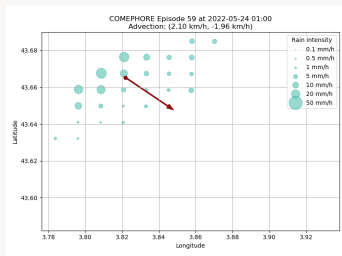
Fusion COMEPHORE-OMSEV \rightarrow more reliable advection



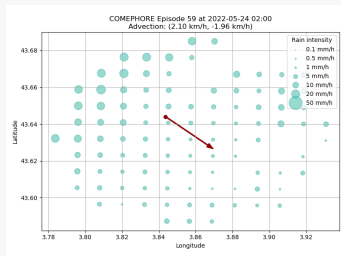
COMEPHORE pixels
Météo France

ADVECTION ESTIMATION ON REAL DATA

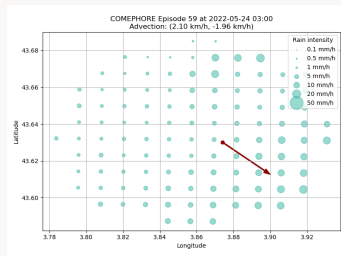
For one episode (COMEPHORE):



$t_0 - 1$



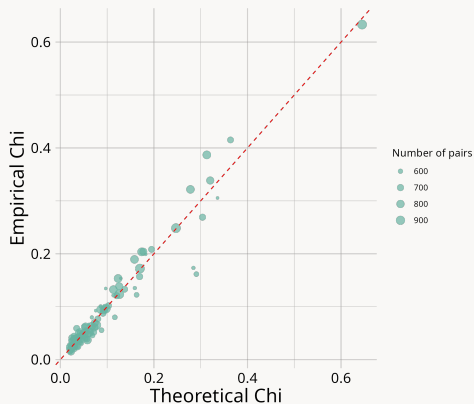
t_0



$t_0 + 1$

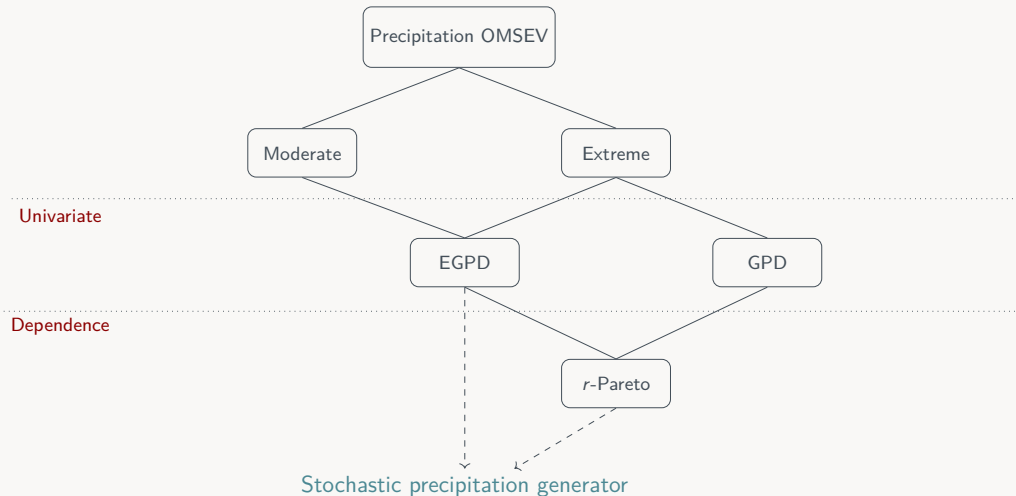
V^{emp} is estimated from **rain storm barycenter displacement** within a time window.

Parameter estimates over $|\mathcal{E}| = 384$ episodes
 $\hat{\eta}_1 = 1.621$ and $\hat{\eta}_2 = 5.219$ fixed (COMEPHORE)

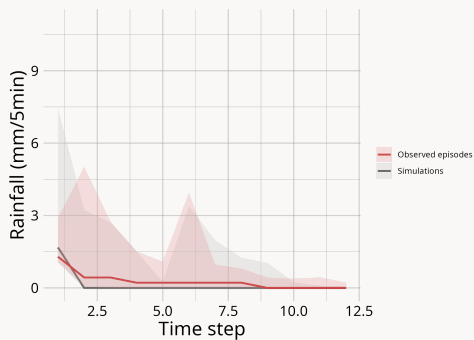


	km/h	m/5 min
$\hat{\beta}_1$	1.090	0.230
$\hat{\beta}_2$	4.628	0.786
$\hat{\alpha}_1$	0.225	0.225
$\hat{\alpha}_2$	0.713	0.713

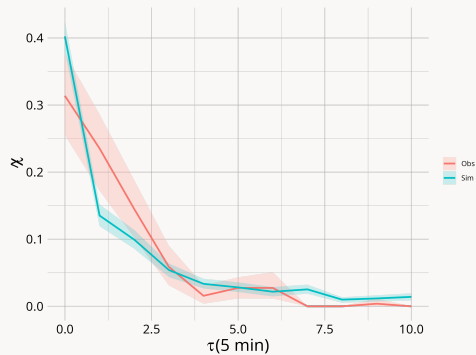
STOCHASTIC SIMULATION OF PRECIPITATION EVENTS



STOCHASTIC SIMULATION OF PRECIPITATION EVENTS



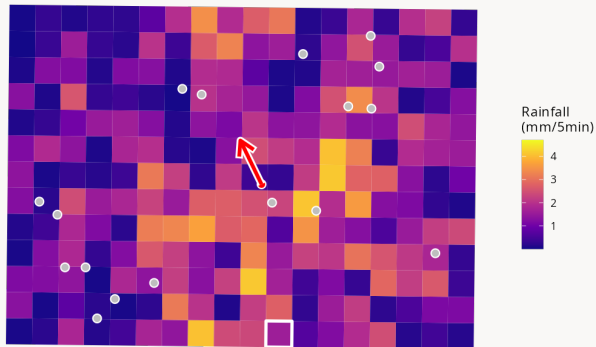
Simulations vs observations at all sites s_0



Temporal r -extremogram for fixed sites

100 simulations with significant advection speed > 1 km/h and South direction

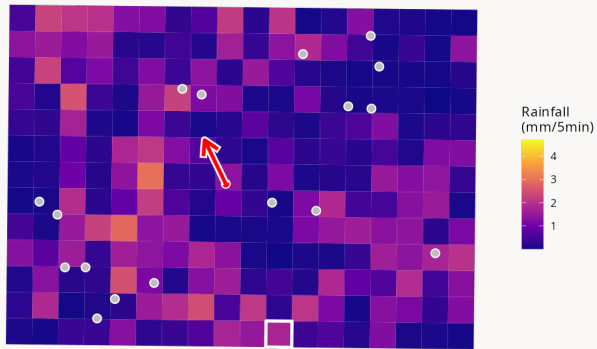
Simulated rainfall field — $t = 1$



Over the OMSEV network area

- ▶ Spatial resolution: $100 \text{ m} \times 100 \text{ m}$
- ▶ Temporal resolution: 5 min
- ▶ Stochastic rainfall simulation

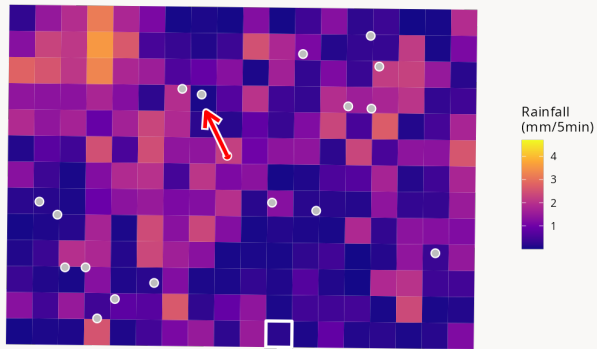
Simulated rainfall field — $t = 2$



Over the OMSEV network area

- ▶ Spatial resolution: $100 \text{ m} \times 100 \text{ m}$
- ▶ Temporal resolution: 5 min
- ▶ Stochastic rainfall simulation

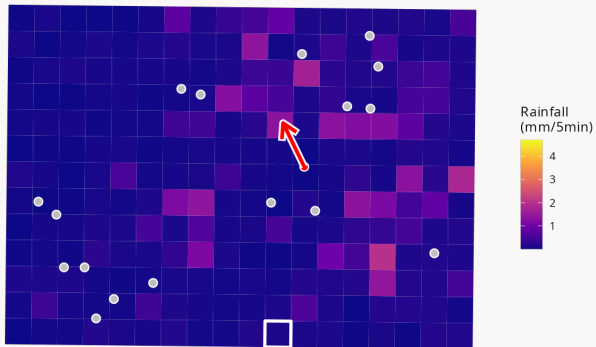
Simulated rainfall field — $t = 3$



Over the OMSEV network area

- ▶ Spatial resolution: $100 \text{ m} \times 100 \text{ m}$
- ▶ Temporal resolution: 5 min
- ▶ Stochastic rainfall simulation

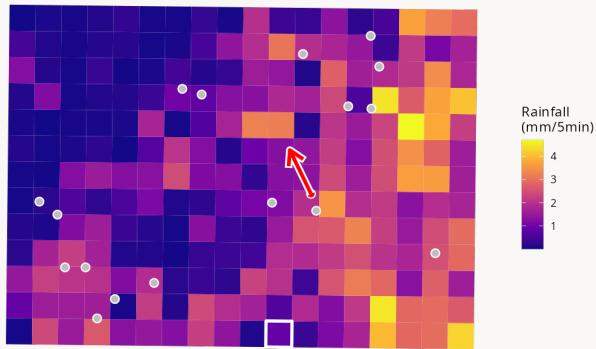
Simulated rainfall field — $t = 4$



Over the OMSEV network area

- ▶ Spatial resolution: $100 \text{ m} \times 100 \text{ m}$
- ▶ Temporal resolution: 5 min
- ▶ Stochastic rainfall simulation

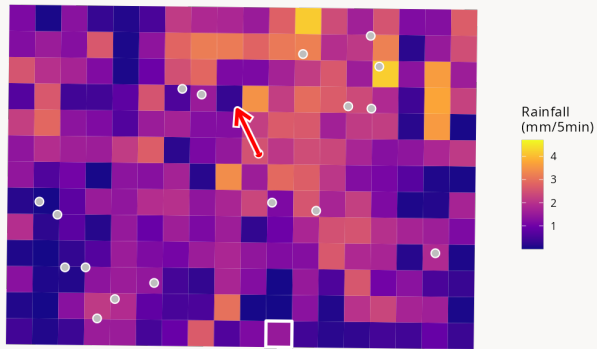
Simulated rainfall field — $t = 5$



Over the OMSEV network area

- ▶ Spatial resolution: $100 \text{ m} \times 100 \text{ m}$
- ▶ Temporal resolution: 5 min
- ▶ Stochastic rainfall simulation

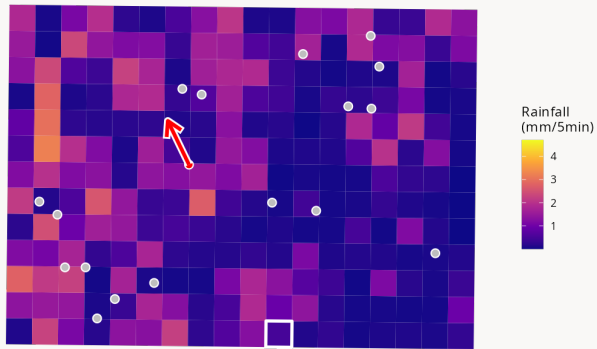
Simulated rainfall field — $t = 6$



Over the OMSEV network area

- ▶ Spatial resolution: $100 \text{ m} \times 100 \text{ m}$
- ▶ Temporal resolution: 5 min
- ▶ Stochastic rainfall simulation

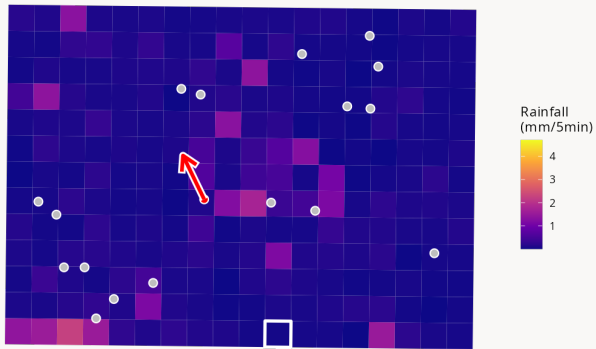
Simulated rainfall field — $t = 7$



Over the OMSEV network area

- ▶ Spatial resolution: $100 \text{ m} \times 100 \text{ m}$
- ▶ Temporal resolution: 5 min
- ▶ Stochastic rainfall simulation

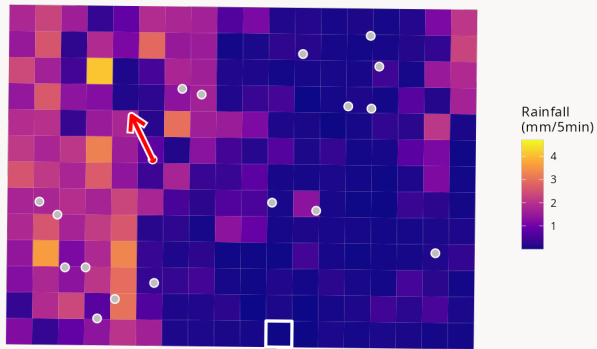
Simulated rainfall field — $t = 8$



Over the OMSEV network area

- ▶ Spatial resolution: $100\text{ m} \times 100\text{ m}$
- ▶ Temporal resolution: 5 min
- ▶ Stochastic rainfall simulation

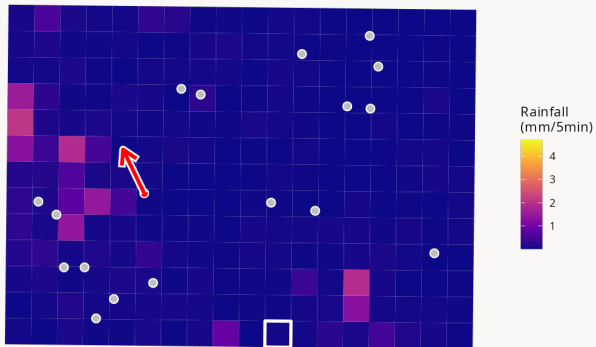
Simulated rainfall field — $t = 9$



Over the OMSEV network area

- ▶ Spatial resolution: $100 \text{ m} \times 100 \text{ m}$
- ▶ Temporal resolution: 5 min
- ▶ Stochastic rainfall simulation

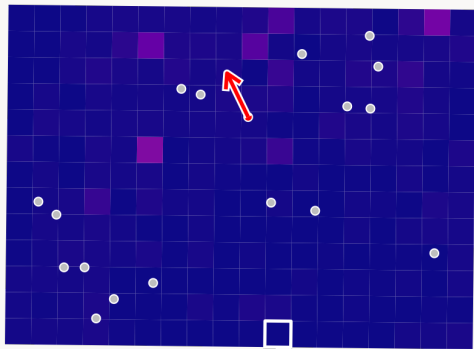
Simulated rainfall field — $t = 10$



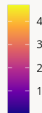
Over the OMSEV network area

- ▶ Spatial resolution: $100\text{ m} \times 100\text{ m}$
- ▶ Temporal resolution: 5 min
- ▶ Stochastic rainfall simulation

Simulated rainfall field — $t = 11$



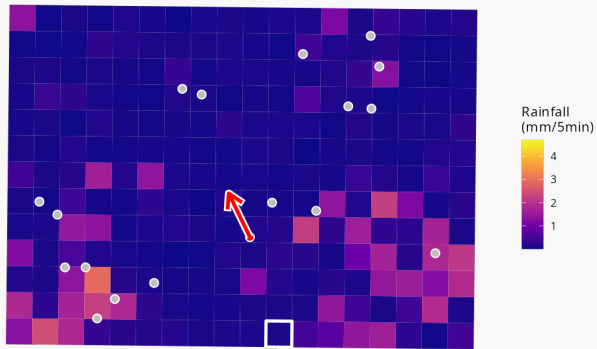
Rainfall
(mm/5min)



Over the OMSEV network area

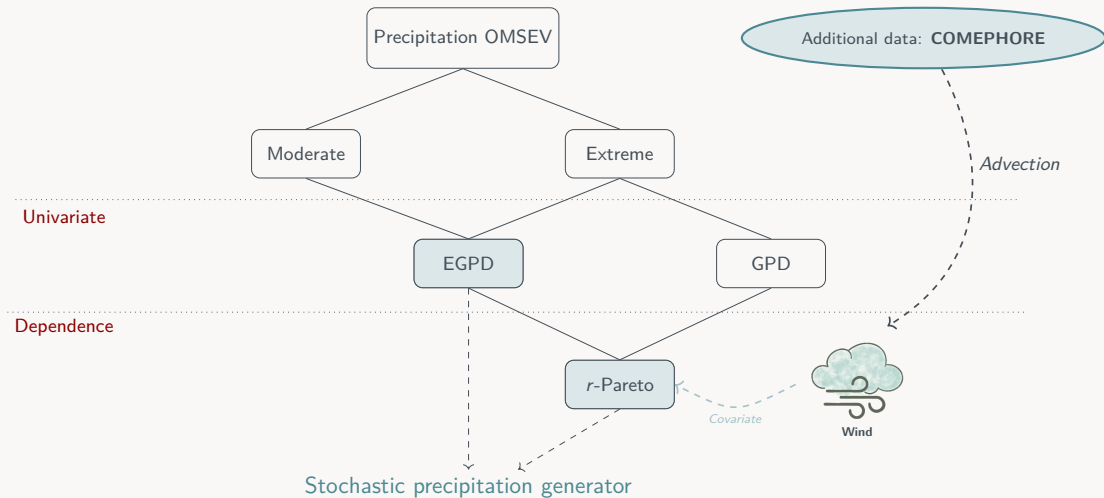
- ▶ Spatial resolution: $100 \text{ m} \times 100 \text{ m}$
- ▶ Temporal resolution: 5 min
- ▶ Stochastic rainfall simulation

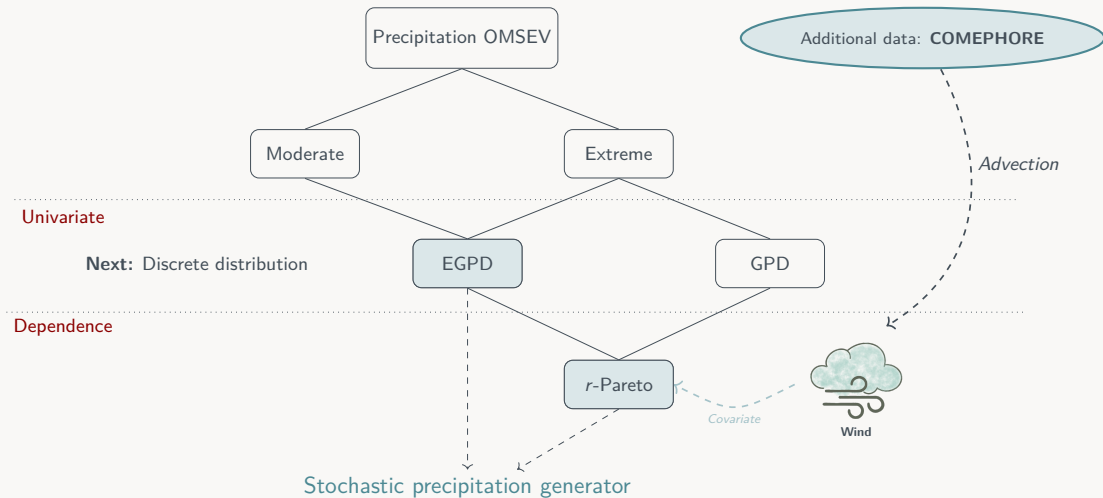
Simulated rainfall field — $t = 12$

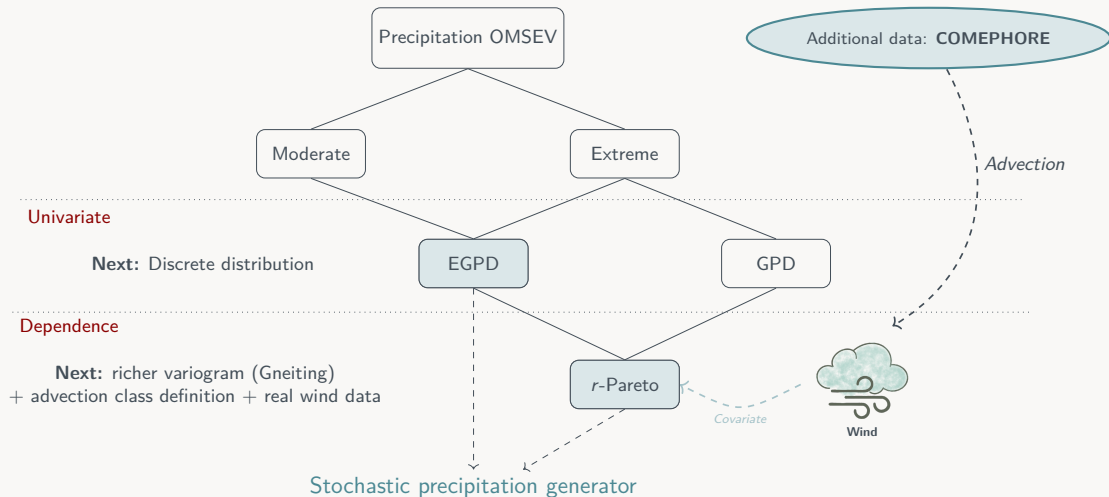


Over the OMSEV network area

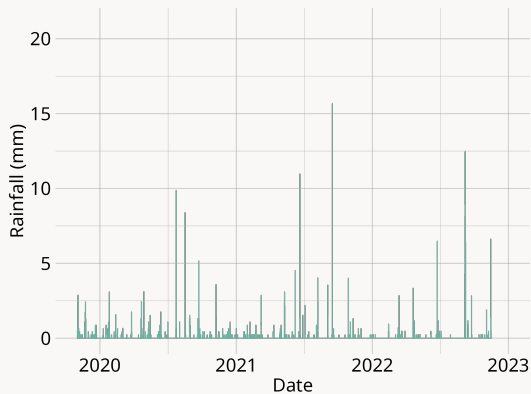
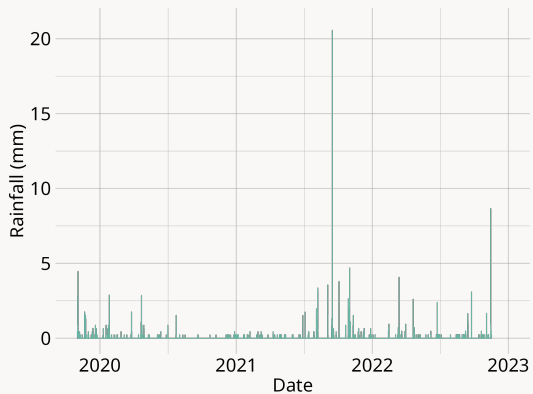
- ▶ Spatial resolution: $100 \text{ m} \times 100 \text{ m}$
- ▶ Temporal resolution: 5 min
- ▶ Stochastic rainfall simulation







RAINFALL DATA - OMSEV



Rainfall amounts on CNRS and Polytech rain gauges

Generalized Pareto Distribution



Extended GPD¹

$$\overline{H}_\xi \left(\frac{x-u}{\sigma} \right) = \begin{cases} (1 + \xi \frac{x-u}{\sigma})_+^{-1/\xi} & \text{if } \xi \neq 0, \\ e^{-\frac{x-u}{\sigma}} & \text{if } \xi = 0, \end{cases}$$

where $a_+ = \max(a, 0)$, $\sigma > 0$, $x - u > 0$

- ▶ Models extreme precipitation
- ▶ Depends on a threshold choice

$$F(x) = G \left(H_\xi \left(\frac{x}{\sigma} \right) \right),$$

where $G(x) = x^\kappa$, $\kappa > 0$

- ▶ Models **moderate and extreme** precipitation
- ▶ Avoids a threshold choice

¹naveau_modeling_2016